Student Name: $\qquad$


Fall 2015
NC Final Exam Advanced Functions and Modeling



1 Suppose the function $H(t)=8.5 \sin (0.017 t-1.35)+12$ models the hours of sunlight for a town in Alaska, where $t=1$ is the first day of the year. Based on the function, what is the approximate range of daylight hours for the town?

A $\quad 3.5$ to 20.5
B $\quad 4$ to 20
C $\quad 4.5$ to 19.5
D $\quad 5$ to 19

2 The lifetime of a particular type of car tire is normally distributed. The mean lifetime is 50,000 miles, with a standard deviation of 5,000 miles. Of a random sample of 15,000 tires, how many of the tires are expected to last for between 45,000 and 55,000 miles?

A 7,125
B 10,200
C 14,250
D 14,850

Advanced Functions and Modeling - Released Items

3 The frequency table below shows the number of runners in specific age groups for a certain race.

| Age Group | Number of <br> Runners |
| :---: | :--- |
| $0-10$ | $\\|$ |
| $11-20$ | $\\|$ |
| $21-30$ | HH HH \\| I |
| $31-40$ | HH I |
| $41-50$ | HH |
| $51-60$ | $\\|$ |
| $61-70$ | $\\|$ |
| $71-80$ | $\\|$ |
| $81-90$ | $\\|$ |

What is the shape of the distribution?
A uniform
B skewed right
C skewed left
D normal

## Advanced functions and Modeling - Released Items

4 A spinner labeled 1 to 9 gives each of the numbers $2,5,7$, and 9 a $\frac{3}{20}$ chance of being landed upon. The chance of landing on each of the other five numbers is equal. If the spinner is spun 1,000 times, which choice is the most likely outcome for the 1,000 spins?

A

| Number on <br> Spinner | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Occurrences | 110 | 112 | 111 | 111 | 109 | 112 | 112 | 111 | 112 |

B

| Number on <br> Spinner | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Occurrences | 82 | 148 | 78 | 80 | 149 | 79 | 151 | 81 | 152 |

C

| Number on <br> Spinner | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Occurrences | 120 | 122 | 100 | 103 | 108 | 126 | 113 | 104 | 104 |

D

| Number on <br> Spinner | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Occurrences | 121 | 100 | 119 | 120 | 102 | 120 | 98 | 121 | 99 |

5 A group of 12 people need to form a line. The line will consist of exactly 9 of the people. Person $X$ and Person $Y$ have to be either third or fourth in line. How many different orders are possible?

A 79,833,600
B 1,209,600
C 604,800
D 362,880

6 The probability that it will rain on Saturday is $\frac{2}{3}$. The probability that the temperature on Saturday will reach $100^{\circ} \mathrm{F}$ is $\frac{4}{9}$. The probability that it will rain or reach $100^{\circ} \mathrm{F}$ on Saturday is $\frac{4}{5}$. What is the probability it will rain and reach $100^{\circ} \mathrm{F}$ on Saturday?

A $\frac{14}{45}$
B $\quad \frac{16}{45}$
C $\quad \frac{24}{45}$
D $\quad \frac{26}{45}$

7 A manufacturing plant produces a special kind of lightbulb.

- Each lightbulb produced has a 0.040 probability of being defective.
- Five lightbulbs are chosen at random from the production line.

To the nearest thousandth, what is the probability that exactly two of the five bulbs will be defective?

A 0.014
B $\quad 0.016$
C 0.018
D 0.020

8 What is the meaning of the base of the function $y=-\log (x)$ ?
A As $y$ decreases by $1, x$ increases by a factor of 10 .
B As $y$ decreases by $1, x$ increases by 10 .
C As $y$ increases by $1, x$ increases by a factor of 10 .
D As $y$ increases by $1, x$ increases by 10.

Advanced functions and Modeling - Released Items

9 The graph of $y=a^{x}$ is shown below.


Which choice is closest to $\log _{a} 3$ ?
A 0.9
B 2.1
C 3.2
D $\quad 4.8$

## AdVanced Functions and Modeling - Released Items

10 A piecewise function is shown below.

$$
h(x)= \begin{cases}-2 x^{2}+5 x+10 & \text { for }-4 \leq x<3 \\ 2 x+3 p & \text { for } 3 \leq x \leq 5\end{cases}
$$

For what value of $p$ will the function be continuous?
A $\frac{10}{3}$

B $\quad \frac{1}{3}$
C $\quad-\frac{25}{3}$
D $\quad \frac{-34}{3}$

11 The equation $y=4.7 x^{\frac{1}{6}}$ is graphed on the coordinate plane. How does increasing the denominator of the exponent transform the graph?

A The transformed graph will approach a horizontal asymptote while the original graph will not.

B The transformed graph will not approach a horizontal asymptote while the original graph will.

C The transformed graph will go to $\infty$ slower than the original graph as the value of $x$ gets larger.

D The transformed graph will go to $\infty$ faster than the original graph as the value of $x$ gets larger.

12 Which function correctly represents the graph below?


A $y=\sin \left(x-\frac{\pi}{3}\right)+4$
B $\quad y=\sin \left(x+\frac{\pi}{3}\right)+4$
C $\quad y=\cos \left(x-\frac{\pi}{3}\right)+4$
D $\quad y=\cos \left(x+\frac{\pi}{3}\right)+4$

13 Which function has an amplitude that is twice the size and a period that is three times the size of the function $y=3 \cos \left(\frac{x}{4}-1\right)+2$ ?

A $\quad y=6 \sin \left(\frac{x}{12}-3\right)+1$
B $\quad y=\frac{3}{2} \cos \left(\frac{3 x}{4}+1\right)-3$
C $\quad y=6 \cos \left(\frac{3 x}{4}-1\right)+3$
D $\quad y=\frac{3}{2} \sin \left(\frac{x}{12}+3\right)-1$

14 A plane takes off and travels at an angle of $40^{\circ}$ north of east at 110 mph for 2 hours. It then adjusts its path to head $10^{\circ}$ west of north and travels in that direction for half an hour at a speed of 100 mph . Approximately how far away is the plane from its starting point?

A 182 miles
B $\quad 200$ miles
C $\quad 238$ miles
D 249 miles

## AdVanced Functions and Modeling - Released Items

15 Which statement is true about the fifth terms of the two sequences below?

$$
\begin{gathered}
a_{n}=3 n^{2}-6 \\
b_{n}=3\left(b_{n-1}-6\right) ; b_{1}=10
\end{gathered}
$$

A The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 63.

B The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 63.

C The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 21.

D The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 21 .

16 Which statement is true about the series shown below?

$$
-4+-2+-1+\frac{-1}{2}+\frac{-1}{4}+\ldots
$$

A The series converges because $|r|<1$.
B $\quad$ The series diverges because $|r|<1$.
C The series converges because $|r|>1$.
D $\quad$ The series diverges because $|r|>1$.

17 What is the explicit form of the equation $a_{n}=a_{n-1}+2(n-1) ; a_{1}=1$ ?
A $\quad a_{n}=2 n-1$
B $\quad a_{n}=n^{2}-n+1$
C $\quad a_{n}=n^{2}-2 n+2$
D $\quad a_{n}=2 n^{2}-2 n-1$

18 An investor bought 1,500 shares of a stock for $\$ 6$ a share. He estimates the probability that the stock will rise to a value of $\$ 25$ a share is $24 \%$, and the probability it will fall to $\$ 2$ a share is $76 \%$. What is the expected value of the investor's profit from buying the stock?

A $\$ 13,560$
B $\$ 9,120$
C $\$ 6,720$
D $\$ 2,280$

## AdVanced Functions and Modeling - Released Items

19 A Ferris wheel is designed in such a way that the height ( $h$ ), in feet, of the seat above the ground at any time, $t$, is modeled by the function
$h(t)=60-55 \sin \left(\frac{\pi}{10} t+\frac{\pi}{2}\right)$.


What is the maximum height a seat reaches?
A 55 feet
B 60 feet
C 110 feet
D 115 feet

20 A teacher counts the final exam as $25 \%$ of each student's class grade. The remaining $75 \%$ is the mean of the student's test scores from the semester. Jaleesa's test scores for the semester are $86,90,92$, and 80 . What is the minimum score Jaleesa must get on the final exam to have a class grade of 85.0 or higher?

A 77
B 79
C 81
D 83

21 Two sides of a triangle measure 10 inches and 13 inches. The included angle between these sides is $55^{\circ}$. What is the approximate measure of the third side of the triangle?

A $\quad 10.9$ inches
B 11.2 inches
C $\quad 13.9$ inches
D $\quad 16.2$ inches

22 The third term of a geometric sequence is 96 , and the fifth term is 1,536 . What is the sum of the first ten terms of this sequence?

A 4,092
B 1,572,864
C 2,097,150
D 33,554,400

This is the end of the Advanced Functions and Modeling Released Items.

## Directions:

1. Look back over your answers for the test questions.
2. Make sure all your answers are entered on the answer sheet. Only what is entered on your answer sheet will be scored.
3. Put all of your papers inside your test book and close the test book.
4. Place your calculator on top of the test book.
5. Stay quietly in your seat until your teacher tells you that testing is finished.
6. Remember, teachers are not allowed to discuss items from the test with you, and you are not allowed to discuss with others any of the test questions or information contained within the test.
