

Released Items

Student Name: Key

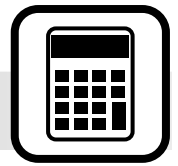
Fall 2015  
NC Final Exam  
**Advanced Functions and  
Modeling**



# Student Booklet



Public Schools of North Carolina  
State Board of Education  
Department of Public Instruction  
Raleigh, North Carolina 27699-6314



1 Suppose the function  $H(t) = 8.5\sin(0.017t - 1.35) + 12$  models the hours of sunlight for a town in Alaska, where  $t = 1$  is the first day of the year. Based on the function, what is the **approximate** range of daylight hours for the town?

- A 3.5 to 20.5
- B 4 to 20
- C 4.5 to 19.5
- D 5 to 19

*Make sure Calc. is in RADIAN MODE*

*Put in  $y =$*

*365 days*

*180 is half way*

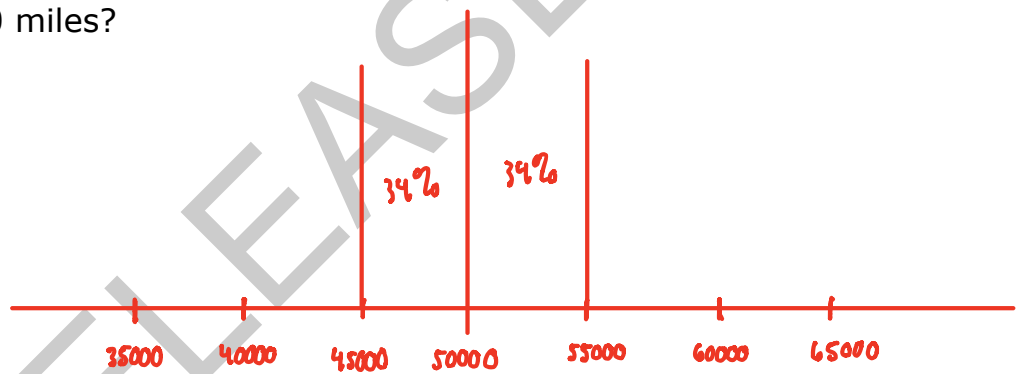
*2nd window*

*change start of table to 180*

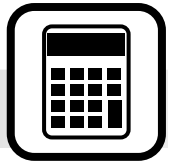
*highest hours of daylight is 20.5*

2 The lifetime of a particular type of car tire is normally distributed. The mean lifetime is 50,000 miles, with a standard deviation of 5,000 miles. Of a random sample of 15,000 tires, how many of the tires are expected to last for between 45,000 and 55,000 miles?

- A 7,125
- B 10,200
- C 14,250
- D 14,850



$68\% \rightarrow .68(15000) = 10,200$

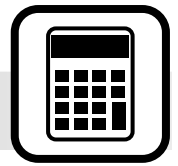


- 3 The frequency table below shows the number of runners in specific age groups for a certain race.

Age Group	Number of Runners
0-10	
11-20	
21-30	
31-40	
41-50	
51-60	
61-70	
71-80	
81-90	

What is the shape of the distribution?

- ~~A~~ uniform
- B skewed right
- ~~C~~ skewed left
- ~~D~~ normal



- 4 A spinner labeled 1 to 9 gives each of the numbers 2, 5, 7, and 9 a  $\frac{3}{20}$  chance of being landed upon. The chance of landing on each of the other five numbers is equal. If the spinner is spun 1,000 times, which choice is the **most likely** outcome for the 1,000 spins?

$$\left(\frac{3}{20}\right)(1000) = 150$$

A

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	110	<del>112</del>	111	111	<del>109</del>	112	<del>112</del>	111	<del>112</del>

B

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	82	148	78	80	149	79	151	81	152

C

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	120	<del>122</del>	100	103	<del>108</del>	126	<del>113</del>	104	<del>104</del>

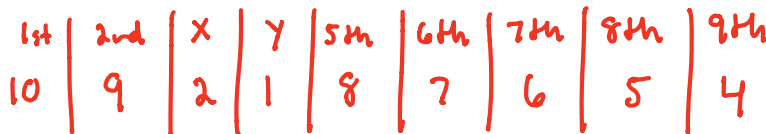
D

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	121	<del>100</del>	119	120	<del>102</del>	120	<del>98</del>	121	<del>99</del>



5 A group of 12 people need to form a line. The line will consist of exactly 9 of the people. Person X and Person Y have to be either third or fourth in line. How many different orders are possible?

- ~~A~~ 79,833,600
- B** 1,209,600
- ~~C~~ 604,800
- ~~D~~ 362,880



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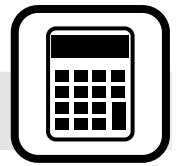

$$\binom{10}{7} \binom{2}{2}$$

6 The probability that it will rain on Saturday is  $\frac{2}{3}$ . The probability that the temperature on Saturday will reach  $100^{\circ}\text{F}$  is  $\frac{4}{9}$ . The probability that it will rain or reach  $100^{\circ}\text{F}$  on Saturday is  $\frac{4}{5}$ . What is the probability it will rain and reach  $100^{\circ}\text{F}$  on Saturday?

- A**  $\frac{14}{45}$
- ~~B~~  $\frac{16}{45}$
- ~~C~~  $\frac{24}{45}$
- ~~D~~  $\frac{26}{45}$

$$\begin{matrix} \text{Rain} & 100^{\circ} & \text{Rain} + 100^{\circ} \\ \left(\frac{2}{3}\right) & + & \left(\frac{4}{9}\right) - \left(\frac{4}{5}\right) = \frac{14}{45} \end{matrix}$$

RELEASED



- 7 A manufacturing plant produces a special kind of lightbulb.
- Each lightbulb produced has a <sup>p</sup>0.040 probability of being defective. q = .96
  - <sup>n</sup>Five lightbulbs are chosen at random from the production line.

To the nearest thousandth, what is the probability that exactly two of the five bulbs will be defective? r

- A 0.014
- B 0.016
- C 0.018
- D 0.020

$$\frac{{}_n C_r (p)^r (q)^{n-r}}{({}_5 C_2) (.04)^2 (.96)^3}$$

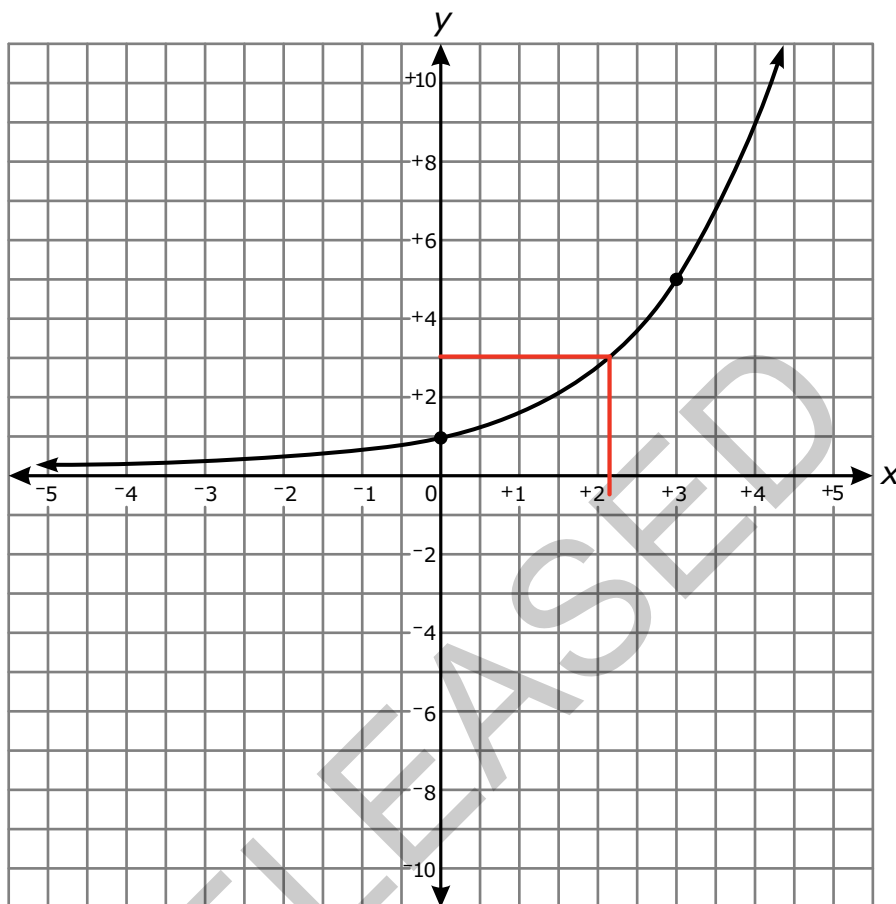
- 8 What is the meaning of the base of the function  $y = -\log(x)$ ?

- A As y decreases by 1, x increases by a factor of 10.
- B As y decreases by 1, x increases by 10.
- C As y increases by 1, x increases by a factor of 10.
- D As y increases by 1, x increases by 10.

plus in y =  
compare x + y  
values



9 The graph of  $y = a^x$  is shown below.



Which choice is closest to  $\log_a 3$ ?

- ~~A~~ 0.9
- B** 2.1
- ~~C~~ 3.2
- ~~D~~ 4.8

$\log_a 3 = x \rightarrow a^x = 3$  (y-value)



10 A piecewise function is shown below.

$$h(x) = \begin{cases} -2x^2 + 5x + 10 & \text{for } -4 \leq x < 3 \\ 2x + 3p & \text{for } 3 \leq x \leq 5 \end{cases}$$

For what value of  $p$  will the function be continuous?

~~A~~  $\frac{10}{3}$

B  $\frac{1}{3}$

~~C~~  $-\frac{25}{3}$

~~D~~  $-\frac{34}{3}$

find  $h(3)$  for each piece

$$-2(3)^2 + 5(3) + 10 = 7$$

$$2(3) + 3p = 7$$

$$6 + 3p = 7$$

$$-6 \quad -6$$

$$3p = 1$$

$$p = \frac{1}{3}$$

11 The equation  $y = 4.7x^{\frac{1}{6}}$  is graphed on the coordinate plane. How does increasing the denominator of the exponent transform the graph?

~~A~~ The transformed graph will approach a horizontal asymptote while the original graph will not.

~~B~~ The transformed graph will not approach a horizontal asymptote while the original graph will.

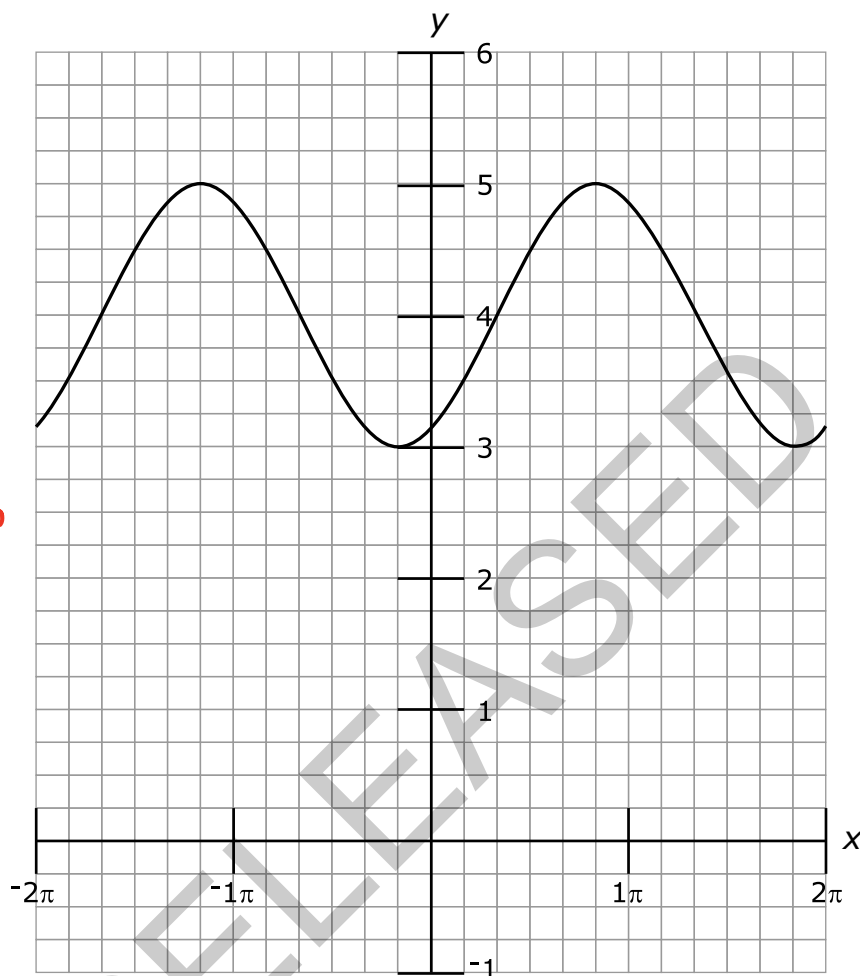
C The transformed graph will go to  $\infty$  slower than the original graph as the value of  $x$  gets larger.

~~D~~ The transformed graph will go to  $\infty$  faster than the original graph as the value of  $x$  gets larger.





12 Which function correctly represents the graph below?



*put calc in  
Radian Mode  
plug answer  
choices into  
y =*

- A  $y = \sin\left(x - \frac{\pi}{3}\right) + 4$
- B  $y = \sin\left(x + \frac{\pi}{3}\right) + 4$
- C  $y = \cos\left(x - \frac{\pi}{3}\right) + 4$
- D  $y = \cos\left(x + \frac{\pi}{3}\right) + 4$



13 Which function has an amplitude that is twice the size and a period that is three times the size of the function  $y = 3 \cos\left(\frac{x}{4} - 1\right) + 2$ ?

A  $y = 6 \sin\left(\frac{x}{12} - 3\right) + 1$

B  $y = \frac{3}{2} \cos\left(\frac{3x}{4} + 1\right) - 3$

C  $y = 6 \cos\left(\frac{3x}{4} - 1\right) + 3$

D  $y = \frac{3}{2} \sin\left(\frac{x}{12} + 3\right) - 1$

$\frac{2\pi}{\left(\frac{1}{4}\right)} = 8\pi \cdot 3 = 24\pi$

$\frac{2\pi}{b} = \frac{24\pi}{1}$

$\frac{24\pi b}{24\pi} = \frac{24\pi}{24\pi}$

$b = 1/12$

14 A plane takes off and travels at an angle of  $40^\circ$  north of east at 110 mph for 2 hours. It then adjusts its path to head  $10^\circ$  west of north and travels in that direction for half an hour at a speed of 100 mph. **Approximately** how far away is the plane from its starting point?

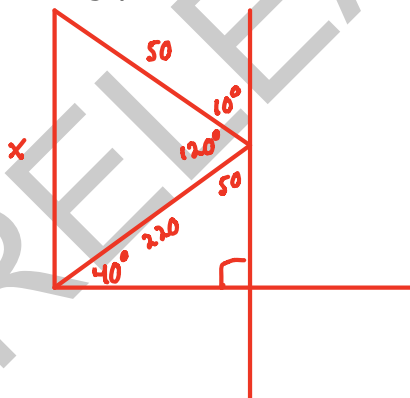
Calc. in DEGREE MODE

A 182 miles

B 200 miles

C 238 miles

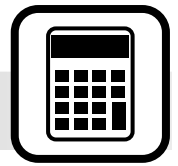
D 249 miles



$x^2 = 50^2 + 220^2 - 2(50)(220) \cos 120$

$\sqrt{x^2} = \sqrt{61900}$

$x \approx 248.7 \text{ mi}$



15 Which statement is true about the fifth terms of the two sequences below?

*Explicit*  $a_n = 3n^2 - 6 = 3(5)^2 - 6 = 69$

$90 - 69 = 21$

*Recursive*  $b_n = 3(b_{n-1} - 6); b_1 = 10$  | 12 | 18 | 36 | 90  
 $b_2$   $b_3$   $b_4$   $b_5$

- A The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 63.
- B The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 63.
- C The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 21.
- D The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 21.

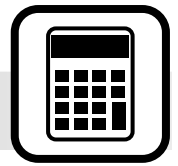
16 Which statement is true about the series shown below?

$$-4 + -2 + -1 + \frac{-1}{2} + \frac{-1}{4} + \dots$$

- A The series converges because  $|r| < 1$ .
- B The series diverges because  $|r| < 1$ .
- C The series converges because  $|r| > 1$ .
- D The series diverges because  $|r| > 1$ .

$r = .5 < 1$

*Convergent*



17 What is the explicit form of the equation  $a_n = a_{n-1} + 2(n - 1)$ ;  $a_1 = 1$ ?

- A  $a_n = 2n - 1$
- B  $a_n = n^2 - n + 1$
- C  $a_n = n^2 - 2n + 2$
- D  $a_n = 2n^2 - 2n - 1$

$$a_2 = 3$$

$$a_3 = 7$$

$$a_4 = 13$$

18 An investor bought 1,500 shares of a stock for \$6 a share. He estimates the probability that the stock will rise to a value of \$25 a share is 24%, and the probability it will fall to \$2 a share is 76%. What is the expected value of the investor's profit from buying the stock?

- A \$13,560
- B \$9,120
- C \$6,720
- D \$2,280

.24	37500 - 9000 = 28500
.76	3000 - 9000 = -6000

$$.24(28,500) + .76(-6,000)$$

$$= \$2,280$$

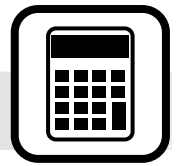
$$1500 \cdot 6 = \$9000$$


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$$1500 \cdot 25 = \$37,500$$


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$$1500 \cdot 2 = \$3000$$



- 19 A Ferris wheel is designed in such a way that the height ( $h$ ), in feet, of the seat above the ground at any time,  $t$ , is modeled by the function

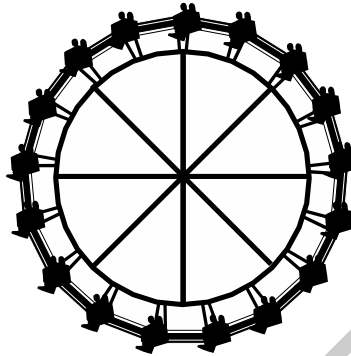
$$h(t) = 60 - 55\sin\left(\frac{\pi}{10}t + \frac{\pi}{2}\right).$$

put Calc. in Radian Mode

put in  $y=$

Look at table

Look for highest  
 $y$ -value



What is the **maximum** height a seat reaches?

- A 55 feet
  - B 60 feet
  - C 110 feet
  - D 115 feet
- 20 A teacher counts the final exam as 25% of each student's class grade. The remaining 75% is the mean of the student's test scores from the semester. Jaleesa's test scores for the semester are 86, 90, 92, and 80. What is the **minimum** score Jaleesa must get on the final exam to have a class grade of 85.0 or higher?

- ~~A~~ 77
- B 79
- ~~C~~ 81
- ~~D~~ 83

find mean

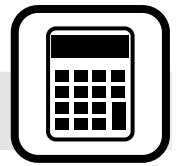
$$\frac{348}{4} = 87$$

$$.75(87) + .25x = 87$$

$$\begin{array}{r} 65.25 + .25x = 87 \\ -65.25 \quad -65.25 \\ \hline .25x = 21.75 \end{array}$$

$$\frac{.25x}{.25} = \frac{21.75}{.25}$$

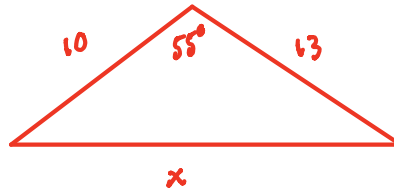
$$x = 87$$



Put Calc. in Degree Mode

- 21 Two sides of a triangle measure 10 inches and 13 inches. The included angle between these sides is  $55^\circ$ . What is the **approximate** measure of the third side of the triangle?

- A 10.9 inches
- B 11.2 inches
- C 13.9 inches
- D 16.2 inches



$$x^2 = 10^2 + 13^2 - 2(10)(13)\cos 55$$

$$\sqrt{x^2} = \sqrt{119.87}$$

$$x \approx 10.94$$

- 22 The third term of a **geometric sequence** is 96, and the fifth term is 1,536. What is the sum of the first ten terms of this sequence?

- A 4,092
- B 1,572,864
- C 2,097,150
- D 33,554,400

$$a_n = a_1(r)^{n-1}$$

$$\frac{1536}{96} = \frac{96r^{5-3}}{96}$$

$$\sqrt{16} = \sqrt{r^2}$$

$$r = 4$$

$$a_n = 6(4)^{n-1}$$

$$\sum_{x=1}^{10} 6(4)^{n-1}$$


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$$96 = a_1(4)^{3-1}$$

$$\frac{96}{16} = \frac{16a_1}{16}$$

$$a_1 = 6$$



**This is the end of the Advanced Functions and Modeling Released Items.**

**Directions:**

- 1. Look back over your answers for the test questions.**
- 2. Make sure all your answers are entered on the answer sheet. Only what is entered on your answer sheet will be scored.**
- 3. Put all of your papers inside your test book and close the test book.**
- 4. Place your calculator on top of the test book.**
- 5. Stay quietly in your seat until your teacher tells you that testing is finished.**
- 6. Remember, teachers are not allowed to discuss items from the test with you, and you are not allowed to discuss with others any of the test questions or information contained within the test.**

RELEASED