

Math Formula Sheet for AFM and Pre-Calculus

Arithmetic Sequence and Series

$$a_n = a_1 + (n - 1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence and Series

$$a_n = a_1 \cdot r^{(n-1)}$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

$$S = \frac{a_1}{1 - r}, \text{ where } |r| < 1$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Conic Sections

Parabola

Focal Length

$$|a| = \frac{1}{4c}$$

Ellipse

Pythagorean Relationship

$$c^2 = a^2 - b^2$$

Hyperbola with

Center (h, k)

Pythagorean Relationship

$$c^2 = a^2 + b^2$$

Foci

$$(h \pm c, k) \text{ or } (h, k \pm c)$$

ADVANCED FUNCTIONS AND MODELING



1. The table below shows the probability distribution of the number of televisions in each house in a community.

Televisions	Probability
0	0.04
1	0.38
2	0.27
3	x
4	y
5 or more	0.13

$$.04 + .38 + .27 + .13 = .82$$

$$1 - .82 = .18 \text{ (x + y combined)}$$

What is the probability that a house in the community will have at least 3 televisions?

$$.18 + .13 = .31$$

- ~~A~~ 0.69
- B 0.31
- ~~C~~ 0.18
- ~~D~~ 0.09

2. Anna and Zach each have \$600 to invest. Anna's investments earn a rate of 10.5%, and Zach's investments earn a rate of 6.5%. **Approximately**, how much more money will Anna have than Zach when Zach's investments are worth \$900? (Assume continuous compounding.) $A = Pe^{rt}$

- ~~A~~ \$184
- ~~B~~ \$241
- C \$255
- ~~D~~ \$264

Zach

$$\frac{900}{600} = \frac{600 e^{.065t}}{600}$$

$$1.5 = e^{.065t}$$

$$\ln 1.5 = \ln e^{.065t}$$

$$\frac{\ln 1.5}{.065} = \frac{.065t}{.065}$$

$$t \approx 6.24$$

Anna

$$A = 600 e^{.105(6.24)}$$

$$A \approx \$1155.32$$

$$1155.32 - 900 = 255.32$$



3. Which statement is true about the sequence shown below?

0, 4.5, 12, 22.5, . . . *series not approaching a value*

- A The series converges because the limit of the sequence as n approaches infinity is infinity.
- B The series converges because the limit of the sequence as n approaches infinity is 30.
- C The series diverges because the limit of the sequence as n approaches infinity is infinity.
- D The series diverges because the limit of the sequence as n approaches infinity is 30.

4. A pharmaceutical company is creating a new cholesterol drug to prevent heart disease. The company must collect data by testing the drug before it is approved. Which would be the **best** method of data collection?

- A experimental study *← researchers try to control as many variables as possible*
- B observational study *← researchers don't manipulate variables*
- C simulation *← uses mathematical formula*
- D survey



5. If the probability of giving birth to a boy is 0.52, what is the **approximate** probability of giving birth to four consecutive boys?

- ~~A~~ 0.021
- ~~B~~ 0.062
- C 0.073
- ~~D~~ 0.130

$$(.52)^4 =$$

6. How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative?

Permutation

- ~~A~~ 94,830
- ~~B~~ 11,628
- ~~C~~ 4,320
- D 4,248

Combination

$$({}_{10}P_4) - ({}_{12}C_5)$$

$$5040 - 792$$

$$= 4248$$

7. A starting line for a hockey team should consist of 3 offensive players, 2 defensive players, and 1 goaltender. A coach has 11 offensive players, 6 defensive players, and 2 goaltenders from which to choose the starting line. How many unique starting lines can the coach create?

- ~~A~~ 132
- ~~B~~ 792
- ~~C~~ 4,950
- D 59,400

$$({}_{11}P_3)({}_6P_2)({}_2P_1)$$

$$59400$$

ADVANCED FUNCTIONS AND MODELING



8. What is the **middle term** for the expansion of $(x^2 + 3)^{12}$?

- A $729x^{12}$
- B $924x^{12}$
- C $673,596x^{12}$
- D $665,280x^{12}$

$r = 6$

$p \quad q$

$$\binom{12}{6} (x^2)^6 (3)^6$$

$$673596 x^{12}$$

9. 15. Abby took an 8-question multiple-choice quiz. Suppose that her probability of correctly answering any question is 0.75. What is Abby's probability of incorrectly answering exactly two questions on the quiz?

- A $P = 0.089$
- B $P = 0.240$
- C $P = 0.311$
- D $P = 0.623$

$p = .75 \quad q = .25$

$$\binom{8}{2} (.25)^2 (.75)^6$$

10. 16. Which function results by shifting the graph of $y = \ln(x + 3) - 6$ to the left 4 units and down 3 units?

- A $y = \ln(x + 7) - 9$
- B $y = \ln(x - 1) - 9$
- C $y = \ln(x + 7) - 3$
- D $y = \ln(x - 1) - 3$

$+4$

$+4 \quad -3$

$$\ln(x+7) - 9$$

ADVANCED FUNCTIONS AND MODELING



11. **A** $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x + 2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$

B $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x - 2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$

C $f(x) = \begin{cases} -2x - 7 & \text{for } x \leq -5 \\ -(x - 2)^2 + 6 & \text{for } -5 < x \leq 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$

D $f(x) = \begin{cases} -2x - 7 & \text{for } x \leq -5 \\ -(x + 2)^2 + 6 & \text{for } -5 < x \leq 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$

12. 18 A function, $f(x)$, is shown below.

$$f(x) = \begin{cases} x - 4 & \text{for } 0 \leq x < 2 \\ x^2 - 3x + 4 & \text{for } 2 \leq x < 4 \\ 5 & \text{for } 4 \leq x < 7 \end{cases}$$

What is the range of $f(x)$?

- ~~A~~ $[-4, 5)$
- B** $[-4, 8)$
- ~~C~~ $[-4, -2) \cup [2, 5)$
- ~~D~~ $[-4, -2) \cup [2, 8)$

Included: []

$$\begin{aligned} (0) - 4 &= -4 \\ (2)^2 - 3(2) + 4 &= 2 \\ 5 & \end{aligned}$$

Not Included: ()

$$\begin{aligned} (2)^2 - 4 &= -2 \\ (4)^2 - 3(4) + 4 &= 8 \end{aligned}$$

$$[-4, -2) \cup [2, 8)$$

ADVANCED FUNCTIONS AND MODELING

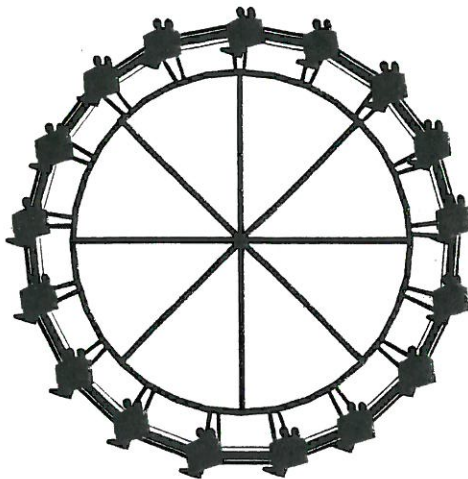


13. A Ferris wheel has a diameter of 80 feet. Riders enter the Ferris wheel at its lowest point, 5 feet above the ground at time $t = 0$ seconds. One complete rotation takes 65 seconds

$r = 40$ ft
vertical shift

amp + 5

= 45



Which function models a rider's vertical height, $h(t)$, at t seconds?

- A $h(t) = -80 \cos\left(\frac{2\pi}{65}t\right) + 5$
- B $h(t) = -40 \cos\left(\frac{2\pi}{65}t\right) + 45$
- C $h(t) = -45 \cos\left(\frac{65}{2\pi}t\right) + 40$
- D $h(t) = -5 \cos\left(\frac{65}{2\pi}t\right) + 80$



14. A bathroom floor has tiles arranged in 9 circles. The innermost circle contains 9 tiles. Each successive circle contains 9 more tiles than the previous circle. How many total tiles are on the bathroom floor?

$$a_1 = 9 \quad n = 9 \quad d = 9$$

~~A~~ 81

$$a_9 = 9 + 9(9-1)$$

~~B~~ 396

$$a_9 = 9 + 72$$

$$S_9 = \frac{9}{2}(9 + 81)$$

C 405

$$a_9 = 81$$

$$S_9 = 4.5(90)$$

~~D~~ 729

$$S_9 = 405$$

This is the end of the multiple-choice portion of the test.

ADVANCED FUNCTIONS AND MODELING



15. Students are told that attending class regularly will help improve their scores in that class. Below are the scores for students who did attend class regularly and scores for those who did not.

Attended Class Regularly					Did Not Attend Class Regularly			
241	261	271	282	296	185	195	195	228
243	262	272	284	296	250	256	225	261
254	267	278	292	308	274	277	308	233
252	264	276	290	310				

- Which group of students has a larger mean score and by how much?
 - Which group of students has a larger median score and by how much?
16. Write an equation for the power function, in $y = ax^b$ form, that passes through the points (2, 1) and (5, 6).
- Use your power function to predict the value of y when $x = 9$.