

**Finding Domain without a Graph**

In the last lesson we found domain and range using the graph. If we do not have the graph to show us undefined values (if any), then we will have to find values that restrict the domain (if any). We will not find range without the graph.

Example 1: Find the domain of  $f(x) = x^2 - 3$

Are there any values of x that will not work in this function?

Domain:  $(-\infty, \infty)$

Example 2: Find the domain of  $f(x) = \frac{1}{(x-4) \neq 0}$   
When would this function be undefined?

*rational function  
Bottom cannot = 0  
 $x \neq 4$*

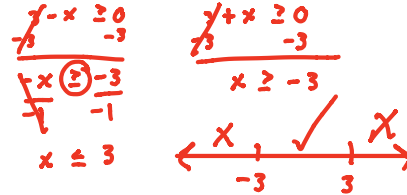
The domain cannot include any undefined values so we exclude those from our domain.

Domain:  $(-\infty, 4) \cup (4, \infty)$

Example 3: Find the domain of  $f(x) = \sqrt{9 - x^2} \geq 0 \rightarrow (3-x)(3+x) \geq 0$

When would this function be undefined?

Domain:  $[-3, 3]$



Rational functions (fractions) have restrictions when there is a variable on the bottom.

How can we find the restrictions? **Always set the denominator equal to zero. This solves for the vertical asymptote(s) and limits of the domain.**

Square Root Functions are undefined when the radicand is less than zero.

How can we find the restrictions? **Set the radicand to  $\geq 0$  and solve.**

More Examples - Find the domain of the following:

4)  $f(x) = \frac{1}{x^2 - x} \neq 0$   
 *$x \neq 0$   $x \neq 1$*   
 $(-\infty, 0) \cup (1, \infty)$

5)  $f(x) = \sqrt{6x - 1} \geq 0$   
 *$6x - 1 \geq 0$   
 $\frac{6x}{6} \geq \frac{1}{6}$   
 $x \geq \frac{1}{6}$*   
 $[\frac{1}{6}, \infty)$

6)  $h(t) = \frac{t}{\sqrt{t+1}} > 0$   
 *$t > -1$*   
 $(-1, \infty)$

## Evaluating Functions

Remember, a variable is just a place holder. So to evaluate a function  $f$  at a number, you substitute the number for the placeholder.

Let  $f(x) = 3x^2 + x - 5$ . Evaluate the function value.

$$(a) f(-2) = 3(-2)^2 + (-2) - 5$$

$$= 5$$

$$(c) f(4) = 3(4)^2 + (4) - 5$$

$$= 47$$

$$(b) f(0) = 3(0)^2 + (0) - 5$$

$$= -5$$

$$(d) f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 5$$

$$= -3.75 \text{ or } -\frac{15}{4}$$

Evaluating a function with new variables will require a lot of simplification.

If  $f(x) = 2x^2 + 3x - 1$ , evaluate the following.

$$(a) f(a) = 2(a)^2 + 3(a) - 1$$

$$= 2a^2 + 3a - 1 \quad \star$$

$$(b) f(-a) = 2(-a)^2 + 3(-a) - 1$$

$$= 2a^2 - 3a - 1$$

$$(c) f(a+h) = 2(a+h)^2 + 3(a+h) - 1$$

$$= 2(a+h)(a+h) + 3a + 3h - 1$$

$2$	$a$	$h$
$a$	$a^2$	$ah$
$h$	$ah$	$h^2$

$$= 2(a^2 + 2ah + h^2) + 3a + 3h - 1$$

$$= 2a^2 + 4ah + 2h^2 + 3a + 3h - 1 \quad \star$$

$$(d) \frac{f(a+h) - f(a)}{h}, h \neq 0$$

$$\frac{2a^2 + 4ah + 2h^2 + 3a + 3h - 1 - (2a^2 + 3a - 1)}{h}$$

$$\frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{3a} + 3h \cancel{-1} - \cancel{2a^2} - \cancel{3a} \cancel{+1}}{h}$$

$$\frac{2h^2 + 4ah + 3h}{h} = 2h + 4a + 3$$