Math 3
Unit 1 Day 2 Notes – Solving Systems

Name: Key
Date: ____________

A system of equations: A set of two or more equations.

A solution is a set of values for the variables that makes all the equations true.

When the "solution" is plugged into the linear equations the result will be a true statement.

3 Ways to solve a system:

1. Graphing
2. Elimination
3. Substitution

# of Solutions	Type of lines
1 →	lines intersect
0 →	parallel lines
infinite →	lines coincide (same line)

Method 1: Graphing

- Solve each equation for y.
- Enter the first equation into Y_1.
- Enter the second equation into Y_2.
- Use the **INTERSECT** option to find where the two graphs intersect (the answer).

2nd TRACE (CALC) #5 intersect

Move spider close to the intersection.

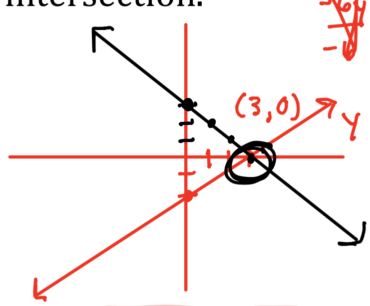
Hit **ENTER** 3 times.

Example #1: $4x - 6y = 12$

$2x + 2y = 6$

$$\begin{aligned} 4x - 6y &= 12 \\ -4x \quad &\quad -4x \\ -6y &= -4x + 12 \\ \tfrac{-6y}{-6} &= \tfrac{-4x}{-6} + \tfrac{12}{-6} \end{aligned}$$

$$\begin{aligned} 2x + 2y &= 6 \\ -2x \quad &\quad -2x \\ \tfrac{2y}{2} &= \tfrac{-2x}{2} + \tfrac{6}{2} \\ y &= -x + 3 \end{aligned}$$



$y = \frac{2}{3}x - 2$

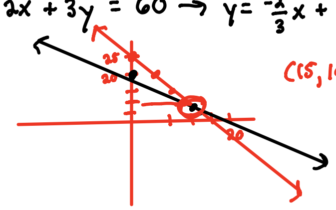
(3,0)

Example #2 Application: There are 25 bikes and trikes at the park. The bikes and trikes have 60 wheels in all. How many bikes and trikes are in the park?

x: bikes (2 wheels)
y: trikes (3 wheels)

$x + y = 25 \rightarrow y = -x + 25$

$2x + 3y = 60 \rightarrow y = -\frac{2}{3}x + 20$



(15,10)

Your try. Solve by graphing. (You can do these by hand or with a calculator!)

1. $-3x + 2y = 8$
$x + 2y = -8$

$y = \frac{3}{2}x + 4$
$y = \frac{-1}{2}x - 4$
$(-4, -2)$

HW 2. $-2x + 4y = 6$
$4x - 8y = 12$

$y = \frac{1}{2}x + 1.5$
$y = \frac{1}{2}x - 1.5$
Same slope
No solution

3. $2x - y = 3$
$6x - 3y = 9$

$y = 2x - 3$
$y = 2x - 3$
Same line
Infinite Solutions

4. Pedro can choose between two tennis courts at two university campuses to learn how to play tennis. One campus charges \$25 per hour. The other campus charges \$20 per hour plus a one –time registration fee of \$10. Write a system of equations to represent the cost c for h hours of court use at each campus. Find the number of hours for which the costs are the same.

$c = 25h$
$c = 20h + 10$

$25h = \cancel{20h} + 10$
$-20h \quad \cancel{-20h}$
$\frac{\cancel{5}h}{\cancel{5}} = \frac{10}{5}$
$h = 2$

Method 2: Algebraically using Elimination

Basic Goal: Add the two equations together so that the x or y is eliminated.

Example #1: $(x - 2y = 14)^{-1}$
$\cancel{x} + 3y = 9$
$\cancel{-x} + 2y = -14$
$\frac{\cancel{5}y}{\cancel{5}} = \frac{-5}{5}$
$y = -1$

$x + 3(-1) = 9$
$x \cancel{-3} = 9$
$\cancel{+3} \quad +3$
$x = 12$
$(12, -1)$

What if the coefficients aren't the same:
No Problem! Follow the steps below.
Basic Steps:
1. Arrange equations so variables, equal signs and constants line up vertically.
2. Multiply one or both equations by a value so that one variable in the 1st equation has the opposite coefficient in the other equation.
3. Add the two equations.
4. Solve for the remaining variable.
5. Use the solution from step 4 and substitute into either equation. Solve for the remaining variable.

Example #2: $(x - 2y = 12)^{6}$
$5y = \cancel{6x} - 23$
$-6x \quad \cancel{-6x}$
$\cancel{-6x} + 5y = -23$
$\cancel{6x} - 12y = 72$
$\frac{\cancel{-7}y}{\cancel{-7}} = \frac{49}{-7}$
$y = -7$

$x - 2(-7) = 12$
$x \cancel{+14} = 12$
$\cancel{-14} \quad -14$
$x = -2$
$(-2, -7)$

Practice with Elimination: Solve using elimination

$x - 2y = 13$ $3x + 2y = 15$ $\frac{4x}{4} = \frac{28}{4}$ $x = 7$ $7 - 2y = 13$ $-7 \quad -7$ $\frac{-2y}{-2} = \frac{6}{-2}$ $y = -3$ $(7, -3)$	$(x - y = 5)^2$ $3x + 2y = 15$ $2x - 2y = 10$ $\frac{5x}{5} = \frac{5}{5}$ $x = 1$ $1 - y = 5$ $-1 \quad -1$ $\frac{-y}{-1} = \frac{4}{-1}$ $y = -4$ $(1, -4)$	$(2x + 8y = 6)^5$ $(-5x - 20y = -15)^2$ $10x + 40y = 30$ $-10x - 40y = -30$ $0 = 0$ Infinite Solutions	$(5x + 4y = -14)^3$ $(3x + 6y = 6)^{-5}$ $15x + 12y = -42$ $-15x - 30y = -30$ $\frac{-18y}{-18} = \frac{-72}{-18}$ $y = 4$ $5x + 4(4) = -14$ $5x + 16 = -14$ $-16 \quad -16$ $\frac{5x}{5} = \frac{-30}{5}$ $x = -6$ $(-6, 4)$

Application: The Algebra 2 classes took 60 minutes to answer a combination of 20 multiple-choice and extended-response questions. The class took 2 minutes to answer each multiple choice question and 6 minutes to answer each extended-response question.

a. Write a system of equations to model the relationship between the number of multiple choice questions **m** and the the number of extended-response questions **r.**

$(m + r = 20)^{-2}$

$2m + 6r = 60$

$-2m - 2r = -40$

$\frac{4r}{4} = \frac{20}{4}$ $\quad r = 5$

$m + 5 = 20$

$-5 \quad -5$

$m = 15$

$(15, 5)$

b. How many of each type of questions was on the test?

$m = 15 \quad r = 5$

Method 3: Substitution

1. Solve one of the equations for either "x =" or "y =".
   This example solves the second equation for "*y*=".
2. Replace the "*y*" value in the first equation by what "*y*" now equals.
3. Solve this new equation for "*x*".
4. Place this new "*x*" value into either of the ORIGINAL equations in order to solve for "*y*".
5. CHECK the solution in BOTH Equations!

Example #1 :

$$2x - 3y = -2$$
$$4x + y = 24$$

$-4x \quad -4x$

$y = -4x + 24$

$2x - 3(-4x + 24) = -2$

$2x + 12x - 72 = -2$

$14x - 72 = -2$

$+72 \quad +72$

$\frac{14x}{14} = \frac{70}{14}$

$x = \boxed{5}$

$y = -4(5) + 24$

$y = -20 + 24$

$y = 4$

$(5, 4)$

Example #2:

$$5x + 8y = 11$$
$$x + 3y = -9$$

$-3y \quad -3y$

$x = -3y - 9$

$5(-3y - 9) + 8y = 11$

$-15y - 45 + 8y = 11$

$-7y - 45 = 11$

$+45 \quad +45$

$\frac{-7y}{-7} = \frac{56}{-7}$

$y = -8$

$x = -3(-8) - 9$

$x = 24 - 9$

$x = 15$

$(15, -8)$

Applications with Systems ~ Pick a Method

Suppose that the Greene Cell Phone company charges $50 per month plus 15 cents per minute while the Johnston Cell Phone Company charges no monthly fee but 25 cents per minute. After how many minutes of phone usage would a monthly phone bill be the same from both companies?

x : minutes

y : cost

$y = .15x + 50$

$y = .25x$

$\cancel{.15x} + 50 = .25x$

$\cancel{-.15x} \quad -.15x$

$\frac{50}{.10} = \frac{\cancel{.10}x}{\cancel{.10}}$

$x = 500$ minutes

Jake's Surf Shop rents surfboards for $6.00 plus $3.00 per hour. Rita's rents them for $9.00 plus $2.50 per hour.

- After how many hours of surfing will the rental fee be the same for both surf shops?

x : hours

y : cost

$y = 3x + 6$

$y = 2.5x + 9$

$3x + 6 = \cancel{2.5x} + 9$

$-2.5x \quad \cancel{-2.5x}$

$.5x + 6 = 9$

$-6 \quad -6$

$\frac{\cancel{.5}x}{\cancel{.5}} = \frac{3}{.5}$

$x = 6$ hours

- You only want to surf for 2 hours; which Surf Shop should you go to?

J: $y = 3(2) + 6$

$12

R: $y = 2.5(2) + 9$

$14

Jack's Surf Shop