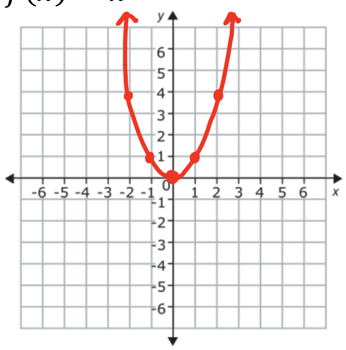
AFM Name Key

Unit 1 Day 3 Notes – Parent Functions & Transformations Date ________

Graph each of the following functions. Use the table to help you come up with quality graphs! Label at least 5 points on your graph.

1. $f(x) = x^2$



x-intercept: (0,0)

y-intercept: (0,0)

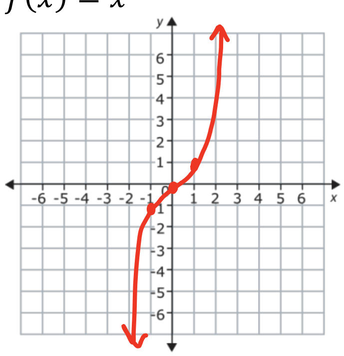
min/max point: (0,0)

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Type of Function: Quadratic

2. $f(x) = x^3$



x-intercept: (0,0)

y-intercept: (0,0)

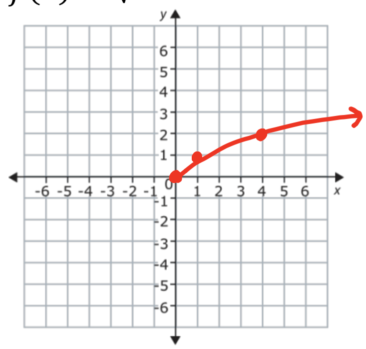
min/max point: N/A

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Type of Function: Cubic

3. $f(x) = \sqrt{x}$



x-intercept: (0,0)

y-intercept: (0,0)

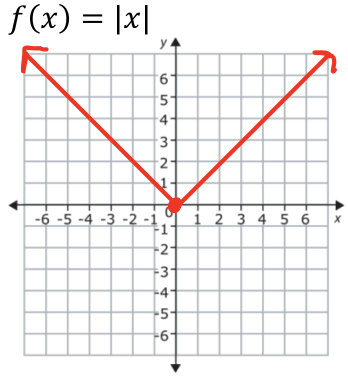
min/max point: (0,0)

Domain: $[0, \infty)$

Range: $[0, \infty)$

Type of Function: Square Root

4. $f(x) = |x|$



x-intercept: (0,0)

y-intercept: (0,0)

min/max point: (0,0)

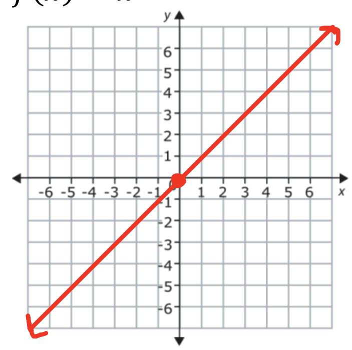
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Type of Function: Absolute Value

$y = mx + b$

5. $f(x) = x$



x-intercept: (0,0)

y-intercept: (0,0)

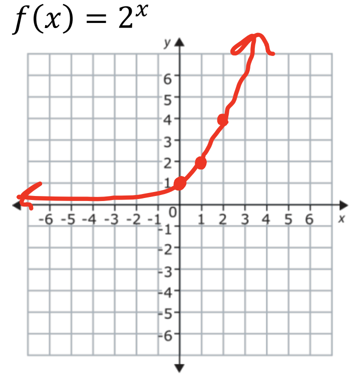
min/max point: N/A

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Type of Function: Linear

6. $f(x) = 2^x$



x-intercept: N/A

y-intercept: (0,1)

min/max point: N/A

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Type of Function: Exponential

These are called **parent** functions or **toolkit** functions. There are several other types but we will use these for our examples.

We will investigate how we can **transform** the parent function to get a new graph. A transformation is a change in a graph from the parent function. A linear transformation moves the graph, but doesn't stretch or shrink the shape.

Transformation Review

Part 1: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$ (Y_1)

a) $y = (x+1)^2$ Y_2

shifted left

b) $y = (x-4)^2$

shifted right

Original #2: $y = |x|$

c) $y = |x+2|$

shifted left

d) $y = |x-5|$

shifted right

e) In one complete sentence, predict what the graph of $y = (x+6)^3$ will look like.

shifted left 6 units

f) In one complete sentence, predict what the graph of $y = \sqrt{x-3}$ will look like.

shifted right 3 units

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x+c)$?

shifts left (+) c units

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x-c)$?

shifts right (-) c units

Inside parentheses shifts graph left or right

Part 2: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$

a) $y = x^2 + 1$

shifts up

b) $y = x^2 - 4$

shifts down

Original #2: $y = |x|$

c) $y = |x| + 2$

shifts up

d) $y = |x| - 5$

shifts down

e) In one complete sentence, predict what the graph of $y = x^3 + 6$ will look like.

shifts up 6 units

f) In one complete sentence, predict what the graph of $y = \sqrt{x} - 3$ will look like.

shifts down 3 units

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) + c$?

shifts up (+) c units

What transformation occurs to the graph of $y = f(x)$ if you have $y = f(x) - c$?

shifts down (-) c units

Outside parentheses shifts up or down

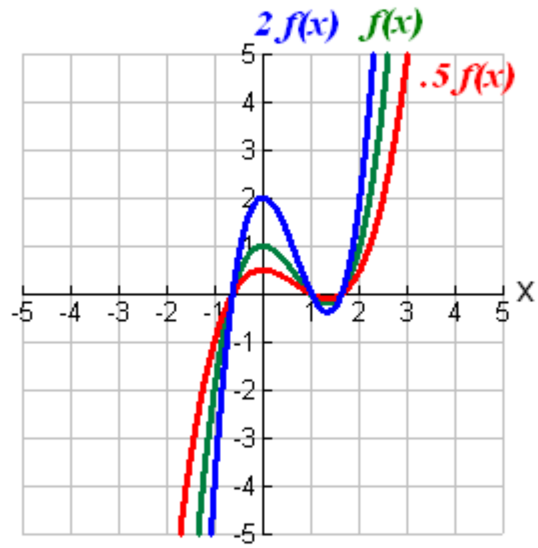
Vertical and Horizontal Stretch and Compression

$a\,f(x)$ stretches/compresses $f(x)$ vertically

A **vertical stretching** is the stretching of the graph away from the x-axis. (graph becomes more narrow)

A **vertical compression (or shrink)** is the squeezing of the graph towards the x-axis. (graph becomes wider)

If the original (parent) function is $y = f(x)$, the vertical stretching or compressing of the function is the function **$a\,f(x)$.**



$f(x) = x^3 - 2x^2 + 1$

Shrink

- if **$0 < a < 1$ (a fraction)**, the graph is **compressed vertically** by a factor of a units.
- if **$a > 1$**, the graph is **stretched vertically** by a factor of a units.
- If **a should be negative,** then the vertical compression or vertical stretching of the graph is followed by a reflection across the x-axis.

Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph? Specifically, look at the 3 original points compared to those 3 "new" points.

Original #1: $y = x^2$ (0,0); (1, 1); (2, 4)

a) $y = 4x^2$

stretch

b) $y = \frac{1}{4}x^2$

shrink

Original #2: $y = |x|$ (0,0); (-1, 1); (2, 2)

c) $y = 2|x|$

stretch

d) $y = -|x|$

reflected

~~e)~~ The points (0,0), (1,1) and (4,2) are on the graph of $y = \sqrt{x}$. What corresponding points do you think will be on the graph of $y = 4\sqrt{x}$?

Generalize: What transformation occurs to the graph of $y = f(x)$ if you have $y = kf(x)$ where k is positive? (Give both cases)

$0 < a < 1$ vertically shrink by a factor of a

$a > 1$ vertically stretch by a factor of a

What transformation occurs to the graph of $y = f(x)$ if you have $y = -f(x)$?

reflected over the x-axis

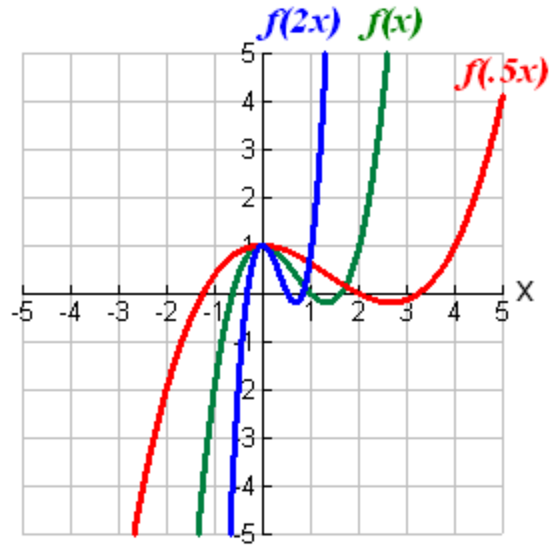
Horizontal Stretch or Compress

$f(ax)$ stretches/compresses $f(x)$ horizontally

A **horizontal stretching** is the stretching of the graph away from the *y*-axis.

A **horizontal compression** is the squeezing of the graph towards the *y*-axis.

If the original (parent) function is $y = f(x)$, the horizontal stretching or compressing of the function is the function $f(ax)$.



$f(x) = x^3 - 2x^2 + 1$

- if $0 < a < 1$ **(a fraction)**, the graph is **stretched horizontally** by a factor of a units.
- if $a > 1$, the graph is **compressed horizontally** by a factor of a units.
- if **a should be negative**, the horizontal compression or horizontal stretching of the graph is followed by a reflection of the graph across the *y*-axis.

Part 4: Look at the graph of $y = \sqrt{x}$ and specifically the points (1,1), (4, 2) and (9, 3).

a) Graph $y = \sqrt{2x}$. Fill in the blanks in these ordered pairs: (____, 1); (____, 2); (____, 3)

b) Now graph $y = \sqrt{4x}$. Fill in the blanks in these ordered pairs: (____, 1); (____, 2); (____, 3)

c) Now graph $y = \sqrt{\frac{1}{2}x}$. Fill in the blanks in these ordered pairs: (____, 1); (____, 2); (____, 3)

<u>**Generalize**</u>: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(kx)$ where k is positive?

$0 < a < 1$ horizontally stretch by a factor of a

$a > 1$ horizontally shrink by a factor of a

$\boxed{\begin{array}{l}\text{Vertical stretch or shrink:} \\ \quad a\,f(x) \rightarrow \text{factor will be in front} \\ \text{Horizontal stretch or shrink:} \\ \quad f(ax) \rightarrow \text{factor will be inside ( )}\end{array}}$

Reflection across the y-axis

Look at the graph of $y = \sqrt{-x}$. Describe the difference between it and the graph of $y = \sqrt{x}$.

a) Compare $y = x^3$ and $y = (-x)^3$. Describe the difference. What happened to the original to get the new graph?

reflected

<u>Generalize</u>: What transformation occurs to the graph of $y = f(x)$ if you have $y = f(-x)$?

reflected over the y-axis

Put It All Together! Without using your calculator, describe the transformations of the graphs. Without graphing, how will the following functions be changed from the parent graph?

a) $f(x) = -|x+1| + 1$

① reflects over the x-axis
② left 1 unit
③ up 1 unit

b) $g(x) = -\sqrt{x+2} - 3$

① reflects over the x-axis
② left 2 units
③ down 3

c) $q(x) = 2x^2 + 4$

① vertical stretch by a factor of 2
② up 4 units

d) $r(x) = 2 - \frac{1}{2}(x-3)^2$

$r(x) = -\frac{1}{2}(x-3)^2 + 2$

① reflected over the x-axis
② vertical shrink by a factor of $\frac{1}{2}$
③ right 3 units
④ up 2 units

Part 7: Think outside the box.

a) Predict what will happen to the graph of $y = \sqrt{-x+1}$. Do not use your calculator yet!

reflect over the y-axis ; shift left 1 units

b) Now, using your calculator graph $y = \sqrt{-x+1}$. What happened to the graph? Were you correct?

reflect over the y-axis ; shift right 1 unit

c) Why do you think the rules applied differently to the graph?

<u>Transformation Rules</u>

Horizontal Transformations:

f(x+c) shifts left

f(x-c) shifts right

Reflections

-f(x) reflects over the x-axis

f(-x) reflects over the y-axis

Vertical Transformations

f(x) + c shifts up

f(x) – c shifts down

Stretch/Shrink

a*f(x) with 0<a<1 vertical shrink

a*f(x) with a>1 vertical stretch

f(a*x) with 0<a<1 horizontal stretch

f(a*x) with a>1 horizontal shrink

Order to perform transformations:
Stretches/reflections, then translations
-or-
All vertical, then all horizontal (or vice-versa)