AFM
Unit 1 Day 4 Notes – Piecewise Functions

Name Key
Date

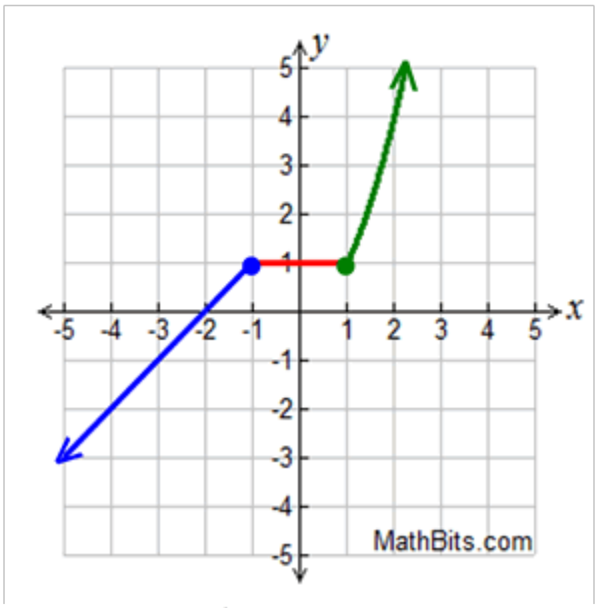
We have seen many graphs that are expressed as single equations and are continuous over a domain of the Real numbers. We have also seen the "discrete" functions which are comprised of separate unconnected "points".

There are also graphs that are defined by "different equations" over different sections of the graphs. These graphs may be continuous, or they may contain "breaks". Because these graphs tend to look like "pieces" glued together to form a graph, they are referred to as "piecewise" functions (*piecewise defined* functions).

Piecewise Function: function defined by two or more different equations applied to different parts of the function

Piecewise defined functions can take on a variety of forms. Their "pieces" may be all linear, or a combination of functional forms (such as constant, linear, quadratic, cubic, square root, cube root, exponential, etc.).

Due to this diversity, there is no "parent function" for piecewise defined functions. The example below will contain linear, quadratic and constant "pieces".



Description:
Notice that the "changes" focus around the *x*-values of 1 and -1.
♦ ***Hint:*** When graphing, focus on where the changes in the graph occur.

From *x*-values of -∞ to -1, the graph is a *straight line*.

From *x*-values of -1 to 1, the graph is *constant*.

From *x*-values 1 to ∞, the graph is *quadratic* (part of a parabola).

$$f(x)=\begin{cases} x+2; & x\le -1 \\ 1; & -1<x<1 \\ x^2; & x\ge 1 \end{cases}$$

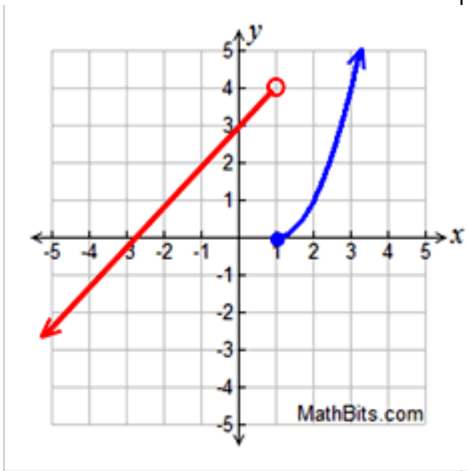
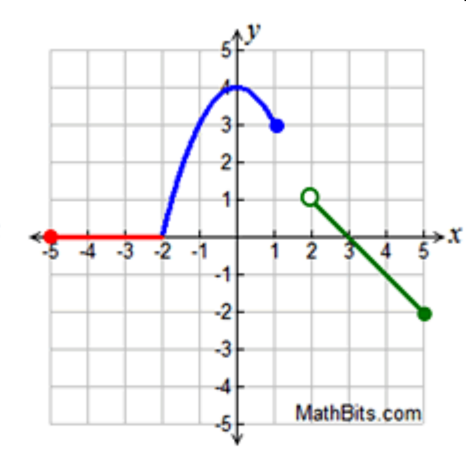
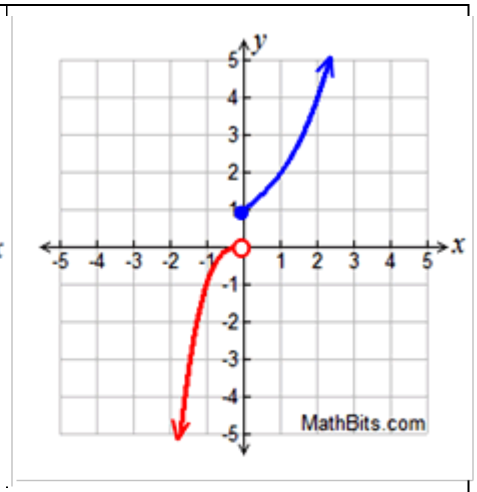
Notice that each "piece" of the function has a specific constraint.

What is the domain of this function?

$(-\infty, \infty)$ or $(-\infty, -1] \cup (-1, 1) \cup [1, \infty)$

Piecewise defined functions may be **continuous** (as seen in the example above), or they may be **discontinuous** (having breaks, jumps, or holes as seen in the examples on the next page).

Other Examples of Piecewise Defined Functions:

		
$f(x)=\begin{cases}(x-1)^2; & x\geq 1\\ x+3; & x<1\end{cases}$	$g(x)=\begin{cases}0; & -5\leq x<-2\\ -x^2+4; & -2\leq x\leq 1\\ -x+3; & 2<x\leq 5\end{cases}$	$f(x)=\begin{cases}2^x; & x\geq 0\\ x^3; & x<0\end{cases}$
Domain: All Reals Range: All Reals	Domain: [-5,1] U (2,5] Range: [-2,4]	Domain: All Reals Range: (-∞,0) U [1,∞)

(From http://mathbitsnotebook.com/Algebra1/FunctionGraphs/FNGTypePiecewise.html)

Evaluating Piecewise Functions

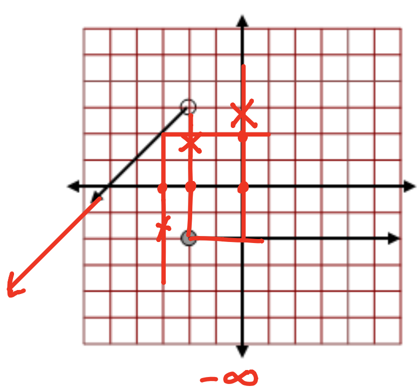
Evaluate the following function at $x=-2, 1, 2, and\ 3$

$$f(x)=\begin{cases}1-x, if\ x\leq 1\\ x^2, if\ x>1\end{cases}$$

a. f(-2) = 1-(-2) = 3   b. f(1) = 1-1 = 0   c. f(2) = $(2)^2$ = 4   d. f(3) = $(3)^2$ = 9

Find the domain and range. Then evaluate the following functions for the given values.

1. (a) Find the domain and range of the graph
   (b) Find the values for h(-2), h(0), h(-3)

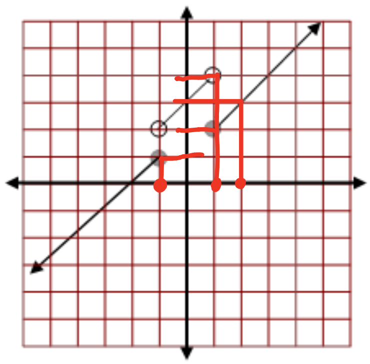


Domain: (-∞,∞) or (-∞,-2) ∪ [-2,∞)

Range: (-∞, 3)

h(-2) = -2   h(0) = -2   h(-3) = 2

2. (a) Find the domain and range of the graph
(b) Find the values for h(-1), h(1), h(2)

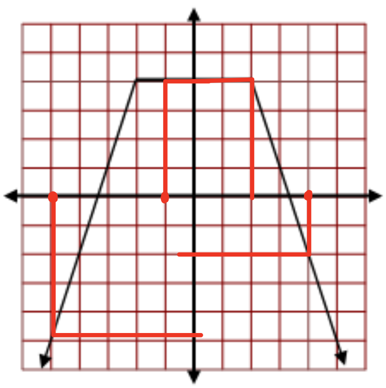


Domain: $(-\infty, \infty)$

Range: $(-\infty, 1] \cup [2, \infty)$

h(-1) = 1 h(1) = 2 h(2) = 3

3. (a) Find the domain and range of the graph
(b) Find the values for h(-5), h(-2), h(2), h(4)



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

h(-5) = -5 h(-2) = 4 h(2) = 4 h(4) = -2

Evaluate the function for the given value of x.

$$f(x) = \begin{cases} 3, & \text{if } x \le 0 \\ 2, & \text{if } x > 0 \end{cases} \qquad g(x) = \begin{cases} x + 5, & \text{if } x \le 3 \\ 2x - 1, & \text{if } x > 3 \end{cases} \qquad h(x) = \begin{cases} \frac{1}{2}x - 4, & \text{if } x \le -2 \\ 3 - 2x, & \text{if } x > -2 \end{cases}$$

1. $f(2)$ **2.** $f(-4)$ **3.** $f(0)$ **4.** $f\left(\frac{1}{2}\right)$

5. $g(7)$ **6.** $g(0)$ **7.** $g(-1)$ **8.** $g(3)$

9. $h(-4)$ **10.** $h(-2)$ **11.** $h(-1)$ **12.** $h(6)$

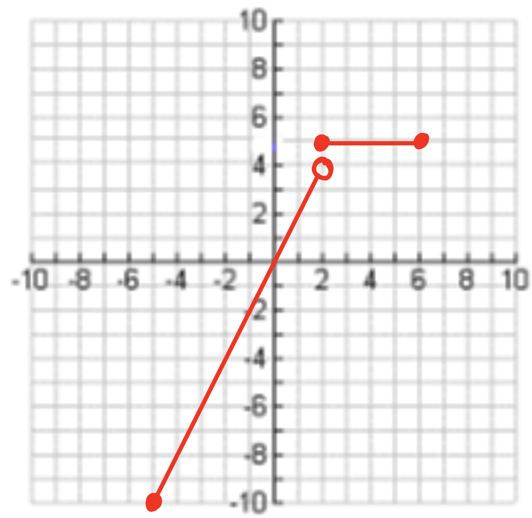
Graphing Piecewise Functions

Both of the following notations can be used to describe a piecewise function over the function's domain:

$$f(x) = \begin{cases} 2x & \text{if } [-5,2) \\ 5 & \text{if } [2,6] \end{cases} \quad \text{or} \quad f(x) = \begin{cases} 2x & , -5 \le x < 2 \\ 5 & , 2 \le x \le 6 \end{cases}$$

1. Complete the following table of values for the piecewise function over the given domain.

x	f(x)
-5	
2	
2	
6	



2. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.

3. How many pieces does your graph have? Why?

4. Are the pieces rays or segments? Why?

5. Are all the endpoints solid dots or open dots or some of each? Why?

6. Were all these x values necessary to graph this piecewise function, or could this have been graphed using less points?

7. Which x values were "critical" to include in order to sketch the graph of this piecewise function?

8. Can you generalize which x-values are essential to input into your table to make a hand sketched graph of a piecewise linear function?

Now graph this piecewise function: $f(x) = \begin{cases} x+3 & ,-8 \le x < 1 \\ 10-2x & ,1 \le x \le 7 \end{cases}$

start (-8, -5) | End (1, 4)

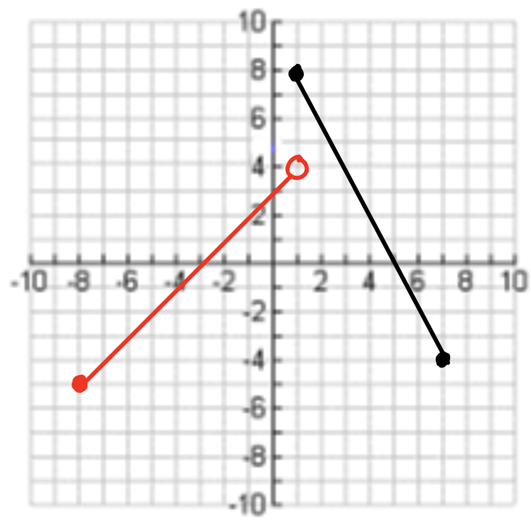
(1, 8) | (7, -4)

Begin by completing a table of values for the piecewise function over the given domain.

x	f(x)

9. Why did you choose the x values you placed into the table?

10. Graph the ordered pairs from your table to sketch the graph of the piecewise function.



11. How many pieces does your graph have? Why?

12. Are the pieces rays or segments? Why?

13. Are all the endpoints filled circles or open circles or some of each? Why?

14. Was it necessary to evaluate both pieces of the function for the x-value 1? Why or why not?

15. Which x values were "critical" to include in order to graph this piecewise function? Explain.

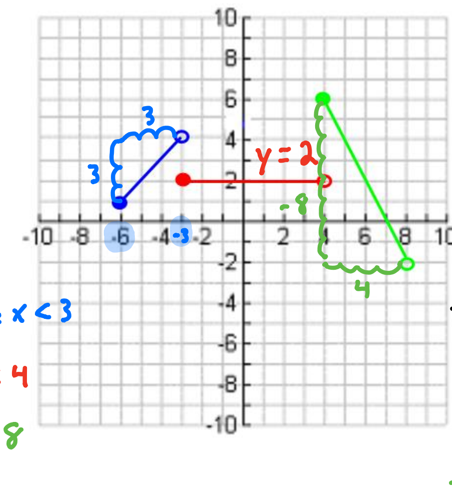
Part 2: Writing piecewise functions given a graph.

16. Can you identify the equations of the lines that contain each segment?

a. Left segment equation= $x + 7$

b. Middle equation= 2

c. Right equation= $-2x + 14$



$y = mx + b$

$(-6, 1)$ $(-3, 4)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{3}{3} = 1 = m$

$4 = (1)(-3) + b$

$4 = -3 + b$

$+3 \quad +3$

$7 = b$ $y = x + 7$

$m = \frac{-8}{4} = -2$ $(8, -2)$

$-2 = -2(8) + b$

$-2 = -16 + b$

$+16 \quad +16$

$b = 14$ $y = -2x + 14$

17. Next, list the domain of each segment.

a. Left segment domain= $[-6, -3)$ or $-6 \le x < 3$

b. Middle domain= $[-3, 4)$ or $-3 \le x < 4$

c. Right domain= $[4, 8)$ or $4 \le x < 8$

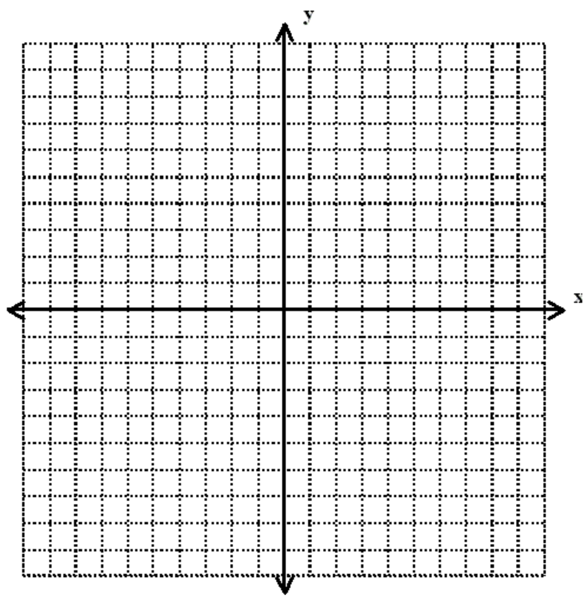
18. Now, put the domain together with the equations to write the piecewise function for the graph.

$$f(x) = \begin{cases} x + 7 & \text{if } [-6, -3) \\ 2 & \text{if } [-3, 4) \\ -2x + 4 & \text{if } [4, 8) \end{cases}$$

HW 19. **Practice:** Graph the following (on the same graph)

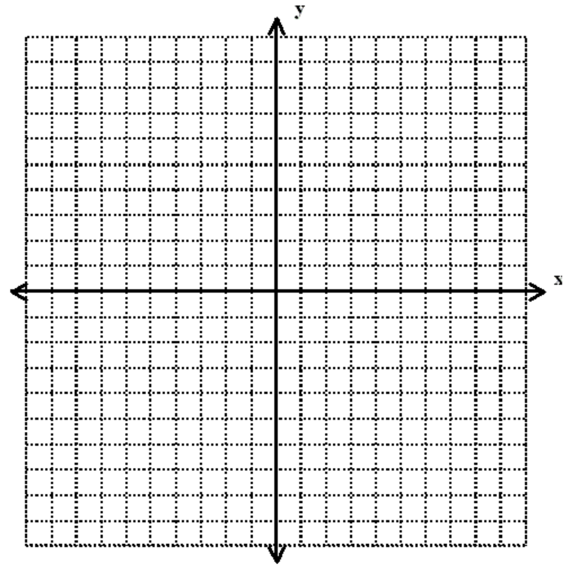
$f(x) = 1 - x, if - 2 \le x \le 1$

$f(x) = x^2, if\ x > 1$



HW

20. Graph $f(x)=\begin{cases}2x-1, \text{if } x\le 1\\ 3x+1, \text{if } x>1\end{cases}$



21. Graph $f(x)=\begin{cases}x^2-1 & x\le 0\\ 2x-1 & 0<x\le 5\\ 3 & x>5\end{cases}$