

Math 3

Name: Key

Unit 1 Day 8 Notes Cont. – Function Operations Date: _____

Review: Evaluate each function for the given value of x.

Let $f(x) = 3x + 4$. Find $f(-2)$.

$$\begin{aligned} f(-2) &= 3(-2) + 4 \\ &= -6 + 4 \\ &= -2 \end{aligned}$$

Let $g(x) = 2x^2 - 3x + 1$. Find $3g(a+2)$

$$\begin{aligned} g(a+2) &= 2(a+2)^2 - 3(a+2) + 1 \\ &= 2(a^2 + 4a + 4) - 3a - 6 + 1 \\ &= 2a^2 + 8a + 8 - 3a - 5 \\ &= 2a^2 + 5a + 3 \end{aligned}$$

$= 6a^2 + 15a + 9$

Part 1: Basic operations with Functions

Operation	Definition	Examples if $f(x) = x + 2$ and $g(x) = 3x$
Sum	$(f + g)x = f(x) + g(x)$	$f(x) + g(x) = (x + 2) + (3x)$ $= 4x + 2$
Difference	$(f - g)x = f(x) - g(x)$	$f(x) - g(x) = (x + 2) - (3x)$ $= (x + 2) - 3x$ $= -2x + 2$
Product	$(f \cdot g)x = f(x) \cdot g(x)$	$f(x) \cdot g(x) = (x + 2)(3x)$ $= 3x^2 + 6x$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$\frac{f(x)}{g(x)} = \frac{(x + 2)}{3x}, x \neq 0$ $\frac{0}{0} = \frac{0}{0}, x \neq 0$

Given $f(x) = x^2 - 3x + 1$ and $g(x) = 4x + 5$, find each function.

a) $(f + g)(x) = f(x) + g(x) = (x^2 - 3x + 1) + (4x + 5) = x^2 + x + 6$

b) $(f - g)(x) = f(x) - g(x) = (x^2 - 3x + 1) - (4x + 5) = x^2 - 7x - 4$

Given $f(x) = x^2 + 5x - 1$ and $g(x) = 3x - 2$, find each function.

a) $(f \cdot g)(x) = (x^2 + 5x - 1)(3x - 2) = 3x^3 + 15x^2 - 13x + 2$

b) $\left(\frac{f}{g}\right)(x) = \frac{(x^2 + 5x - 1)}{(3x - 2)}, x \neq \frac{2}{3}$

Composition of Functions—taking the output (y-value) of one function and making it the input (x-value) of another function.



Consider this

You have a coupon for \$25 off at Hollister.

There is also a 30% off sale for the Jeans that you want to buy.

↳ .3

Does the order in which these discounts are taken matter?

Original Price	1st Discount	2nd Discount	Sale Price
\$50	$\$50 \cdot .3 = \$15 \rightarrow \$35$	$\$35 - \$25 =$	\$10
\$50	$\$50 - \$25 = \$25$	$\$25 \cdot .3 = \7.50	\$17.50

So it is better for the shopper to _____.

Remember

Domain: the set of all input values

Range: the set of all output values

Definition of Composition of Functions: The composition of function f with function g is written

$$f \circ g(x) = f(g(x)).$$

DOES NOT MEAN
MULTIPLICATION

START ON THE INSIDE & WORK YOUR WAY OUT!!!

Let's Watch a Video: <http://www.youtube.com/watch?v=S4AEZEITPDo>

$$f(x) = x^2 + x$$

$$g(x) = 4 - x$$

Find $(f \circ g)(x) = f(g(x))$

$$= (4 - x)^2 + (4 - x)$$

$$= (4 - x)(4 - x) + 4 - x$$

m	4	-x
4	16	-4x
-x	-4x	x ²

$$= x^2 - 8x + 16 + 4 - x$$

$$= x^2 - 9x + 20$$

Find $(g \circ f)(x) = g(f(x))$

$$= 4 - (x^2 + x)$$

$$= 4 - x^2 - x$$

$$= -x^2 - x + 4$$

I. Finding a value of a composition given a function

Given $f(x) = x + 5$ and $g(x) = x^2 - 2$. Evaluate each expression.

a. $f(g(3)) =$

$$g(3) = (3)^2 - 2 = 7$$

$$f(7) = (7) + 5 = 12$$

b. $g(f(3)) =$

$$f(3) = (3) + 5 = 8$$

$$g(8) = (8)^2 - 2 = 62$$

II. Finding a composition equation given functions

Given $f(x) = 3x - 2$ and $g(x) = -2x + 4$

Find $f \circ g(x) = f(g(x))$

$$= 3(-2x + 4) - 2$$

$$= -6x + 12 - 2$$

$$= -6x + 10$$

Find $g \circ f(x) = g(f(x))$

$$= -2(3x - 2) + 4$$

$$= -6x + 4 + 4$$

$$= -6x + 8$$

REMEMBER!!!
 $f \circ g(x) = f(g(x))$
 NOT
 $f(x)g(x)$
 Composing is
 NOT
 MULTIPLICATION!

Function Composition Practice!

1. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for $f(x) = x + 3$ and $g(x) = x^2 + x - 1$.

$$f(g(x)) = (x^2 + x - 1) + 3$$

$$= x^2 + x + 2$$

$$g(f(x)) = (x + 3)^2 + (x + 3) - 1$$

$$= (x + 3)(x + 3) + x + 3 - 1$$

m	x	3
x	x ²	3x
3	3x	9

2. Evaluate $(f \circ g)(x)$ and $(g \circ f)(x)$ for $x = 2$.

$$f(g(2)) = (2)^2 + (2) + 2$$

$$= 4 + 4$$

$$= 8$$

$$g(f(2)) = (2)^2 + 7(2) + 11$$

$$= 4 + 14 + 11$$

$$= 29$$

$$= x^2 + 6x + 9 + x + 2$$

$$= x^2 + 7x + 11$$

3. Find $(g \circ h)(x)$ and $(h \circ g)(x)$ if $g(x) = 2x$ and $h(x) = x^3 + x^2 + x + 1$

$$\begin{aligned} (g \circ h)(x) &= 2(x^3 + x^2 + x + 1) \\ &= 2x^3 + 2x^2 + 2x + 2 \end{aligned}$$

$$\begin{aligned} (h \circ g)(x) &= (2x)^3 + (2x)^2 + (2x) + 1 \\ &= 8x^3 + 4x^2 + 2x + 1 \end{aligned}$$

4. If $f(x) = x^2 - x$ and $g(x) = x - 1$, what is $f(g(x))$?

$$\begin{aligned} f(g(x)) &= (x-1)^2 - (x-1) \\ &= x^2 - 2x + 1 - x + 1 \\ &= x^2 - 3x + 2 \end{aligned}$$

5. A box of laundry detergent sells for \$3.25. The price the store pays is determined by the function $f(x) = x - 1$, where x is the selling price of the detergent. The wholesale price is determined by $g(x) = x - .75$, where x is the price the store pays. What is the wholesale price?

APPLICATIONS

6. A store is offering a 10% discount on all items. In addition, employees get a 25% discount.

a. Write a composite function to model taking the 10% discount first.

b. Write a composite function taking the 25% discount first.

c. Suppose you are an employee. Which discount would you prefer taken first?

7. Tyrone Davis has \$180 deducted from every paycheck for retirement. He can have these deductions taken before taxes are applied, which reduces his taxable income. His federal income tax rate is 18%. If Tyrone earns \$2200 every pay period, find the difference in his net income if he has the retirement deduction taken before taxes or after taxes.

BIG SALE

