

Things to recall from previous courses about right triangles:

- The angles of a triangle add up to 180 degrees.
- One angle is 90 degrees, therefore leaving the other angles to be acute (less than 90 degrees). Why? Because if the angles add up to 180 and one is 90, the other two have to be less than 90 together or otherwise it would be greater than 180 degrees.
- The side across from the right angle is called the hypotenuse. This is always the longest side
- θ = theta, the measure of an angle (could be degrees or radians but we will use degrees for right now).
- If you know 2 sides of a right triangle, you can find the other using the Pythagorean Theorem. $a^2 + b^2 = c^2$ (Whatever it equals squared must be the hypotenuse. It is NOT necessarily side c each time in each triangle, it just depends on how it is labeled)
- Each angle is represented with a capital letter and its corresponding side is represented by the lower case letter of that.

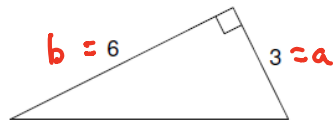
Pythagorean Theorem

- a. Pythagorean Theorem is used to find missing sides in a triangle.

$$a^2 + b^2 = c^2$$

- b. "a" and "b" represent the legs of a right Δ
- c. "c" represents the hypotenuse (longest side in a right Δ)
- d. Examples: Find the missing sides using Pythagorean Theorem

i.

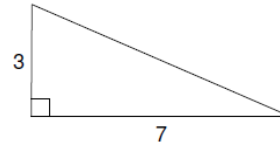


$$3^2 + 6^2 = c^2$$

$$\sqrt{45} = \sqrt{c^2}$$

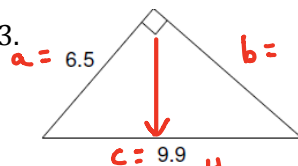
$$c \approx 6.71$$

2.



$$c \approx 7.62$$

3.



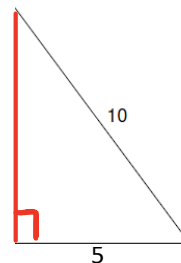
$$\cancel{6.5^2} + b^2 = 9.9^2$$

$$-6.5^2 \quad -6.5^2$$

$$\sqrt{b^2} = \sqrt{55.76}$$

$$b \approx 7.47$$

4.

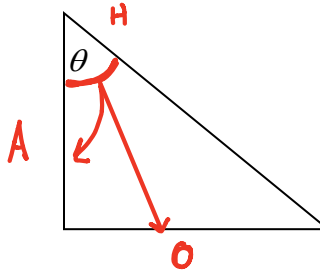
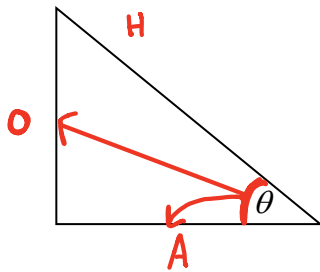


$$b = 8.66$$

Labeling Triangles

In a triangle, the location of theta determines the location of our opposite and adjacent sides. Opposite – straight across from (opposite of it), and adjacent – right next to. (Hypotenuse is always the side opposite the right angle.)

Ex:



SOHCAHTOA

SOHCAHTOA is used to help find missing sides and angles in a right triangle when Pythagorean Theorem does not work!

S (sine) O (opposite) H (hypotenuse)

$$\sin \theta = \frac{\text{opp.}}{\text{hyp.}}$$

C (cosine) A (adjacent) H (hypotenuse) →

$$\cos \theta = \frac{\text{adj.}}{\text{hyp.}}$$

T (tangent) O (opposite) A (adjacent) →

$$\tan \theta = \frac{\text{opp.}}{\text{adj.}}$$

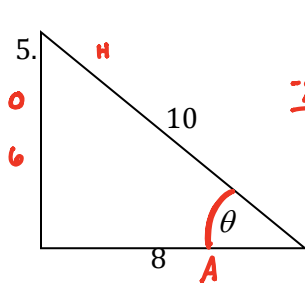
Inverse Trig. Function

$$\text{CSC} = \frac{\text{hyp.}}{\text{opp.}} \quad (\sin)$$

$$\text{SEC} = \frac{\text{hyp.}}{\text{adj.}} \quad (\cos)$$

$$\text{COT} = \frac{\text{adj.}}{\text{opp.}} \quad (\tan)$$

Given the following, find the six trig ratios. (This means, don't find angles, just set up the sides based on the ratios under SOH-CAH-TOA).



$$\begin{aligned} x^2 + x^2 &= 10^2 \\ -x^2 & \quad -x^2 \\ \hline 2x^2 &= 100 \\ x^2 &= 50 \\ x &= 7.07 \end{aligned}$$

$$\sin \theta = \frac{6}{10} = \frac{3}{5}$$

$$\csc \theta = \frac{5}{3}$$

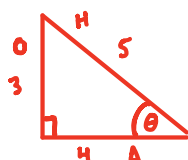
$$\cos \theta = \frac{8}{10} = \frac{4}{5}$$

$$\sec \theta = \frac{5}{4}$$

$$\tan \theta = \frac{6}{8} = \frac{3}{4}$$

$$\cot \theta = \frac{4}{3}$$

6. Find all 6 trig ratios if $\sin \theta = \frac{3}{5}$



$$\begin{aligned} x^2 + x^2 &= 5^2 \\ -x^2 & \quad -x^2 \\ \hline 2x^2 &= 25 \\ x^2 &= 12.5 \\ x &= 3.54 \end{aligned}$$

* reduce fractions if necessary

$$\sin \theta = \frac{3}{5}$$

$$\csc \theta = \frac{5}{3}$$

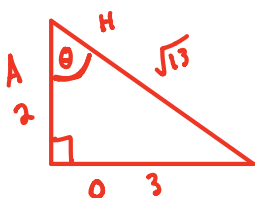
$$\cos \theta = \frac{4}{5}$$

$$\sec \theta = \frac{5}{4}$$

$$\tan \theta = \frac{3}{4}$$

$$\cot \theta = \frac{4}{3}$$

7. Find all 6 trig ratios if $\cot \theta = \frac{2}{3}$



$$\begin{aligned} 2^2 + 3^2 &= c^2 \\ \sqrt{13} &= \sqrt{c^2} \end{aligned}$$

$$\sin \theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\csc \theta = \frac{\sqrt{13}}{3}$$

$$\cos \theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\sec \theta = \frac{\sqrt{13}}{2}$$

$$\tan \theta = \frac{3}{2}$$

$$\cot \theta = \frac{2}{3}$$

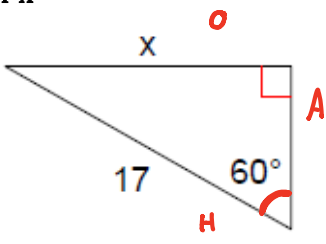
Finding Missing Sides of Triangles

Setting up Trigonometry Ratios and Solving for Sides

- Identify the \angle (NOT the right angle)
- Label the Δ (Opposite, Adjacent, Hypotenuse)
- Figure out trig function
 - ✓ sin if we have the opposite and hypotenuse
 - ✓ cos if we have the adjacent and the hypotenuse
 - ✓ tan if we have the opposite and the adjacent
- iv. Set up the proportion and solve for x!

Example: Solve for x

8.



SOH CAH TOA

$$\frac{\sin 60^\circ}{1} = \frac{x}{17}$$

$$x = 17 \sin 60$$

$$x \approx 14.72$$

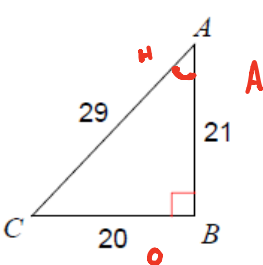
Finding Missing Angles of Triangles

Setting up Trigonometry Ratios and Solving for Angles

- Select a given angle (NOT the right angle)
- Label your sides (Opposite, Adjacent, Hypotenuse)
- Decide which trig function you can use:
 - ✓ SOH if we have the opposite and hypotenuse
 - ✓ CAH if we have the adjacent and the hypotenuse
 - ✓ TOA if we have the opposite and the adjacent
- iv. Solve the equation ... You will need to use an inverse trig function to solve equation!

Example: Find the measure of angle A

9.



SOH CAH TOA

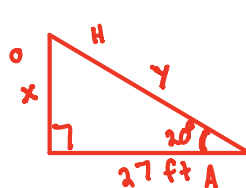
$$\sin^{-1} A = \frac{20}{29}$$

$$\cos^{-1} A = \frac{21}{29}$$

$$\tan^{-1} = \frac{21}{20}$$

$$A = 44^\circ$$

10. A power line snaps in half because of a tornado. It breaks into 2 pieces and forms a right angle with the ground. The top of the power line rests 27 feet from the base of the pole and forms a 20° angle with the ground. Find the original height of the power line before the storm.



SOH CAH TOA

$$\frac{\tan 20^\circ}{1} = \frac{x}{27}$$

$$x = 27 \tan 20$$

$$x \approx 9.83 \text{ ft}$$

$$\frac{\cos 20^\circ}{1} = \frac{27}{y}$$

$$\frac{y \cos 20^\circ}{\cos 20^\circ} = \frac{27}{\cos 20^\circ}$$

$$y \approx 28.7 \text{ ft}$$

$$28.7 + 9.83 = 38.53 \text{ ft.}$$