Math 3 Name: Key

Unit 2 Day 2 Notes - Exponential Growth & Decay Date:_____

GROWTH Scenario: There has been a zombie invasion. The number of zombies increases by 45% each hour. If 3 zombies initially rolled into town at midnight, how many zombies will there be by 30 minutes past noon?

1 + decimal

a = 3 b = 1 + .45

x = 12.5 hours b = 1.45

Step 1: Create a table for the scenario. Start with x=0

X	0	1	2	3	4
Y	3	4.35 → 5	6	9	13

x1.45 x1.45

Step 2: Write a recursive (NOW-NEXT) equation for the scenario:

Note: *The common ratio is the PERCENTAGE (written as a decimal) remaining after one time period has gone by.*

Next = Now • 1.45

starts @ 3

Step 3: Write an explicit equation for the scenario: $y = a(b)^x$

Note: *All exponential equations are in the form $y=a*b^x$. a = initial value, b = common ratio*

$y = 3(1.45)^x$

Step 4: x = the amount of time (or time periods) that have gone by. Choose/substitute an x in order to solve the question.

$y = 3(1.45)^{12.5}$

Answer: 312 zombies

DECAY scenario: The zombie invasion is wiping out the population. The number of normal people are diminishing fast. Each day that goes by 48% of the living population is lost. If the population of North Carolina started out at 9.752 million, how many people will be left after one week?

1 − decimal

↳ a b = 1 − .48 = .52

Step 1: Create a table for the scenario. Start with x=0

X	0	1	2	3	4
Y (millions →)	9.752	5.071	2.636	1.37	.712

x.52 x.52 x.52

Step 2: Write a recursive (NOW-NEXT) equation for the scenario:

Note: *The common ratio is the PERCENTAGE (written as a decimal) remaining after one time period has gone by.*

start @ 9.752

Next = Now • .52

Step 3: Write an explicit equation for the scenario: $y = a(b)^x$

Note: *All exponential equations are in the form $y=a*b^x$. a = initial value, b = common ratio*

$y = 9.752(.52)^x$

Step 4: x = the amount of time (or time periods) that have gone by. Choose/substitute an x in order to solve the question.

one week → 7 days

$y = 9.752(.52)^7$

Answer: .1 millions

Note:

- For growth scenarios → the common ratio is greater than 1. → Can be found by doing 100% + (% of increase) then write it as a decimal.
- For decay scenarios → the common ratio is less than 1. → Can be written as 100% - (% of decrease) then write it as a decimal.

Special Circumstances

Compound Interest Scenario: Mary places $5000 into a savings account that earns 3.1% interest compounded quarterly. How much money will Mary have in her account after 15 years?

*NOTE: Compound Interest is a special type of GROWTH scenario. To calculate the common ratio:
1 + (% interest written as a decimal / # of times compounded per year)
Additionally, x (amount of time) must be multiplied by the # of times compounded per year.
Therefore, your final equation looks like:

$y = a(1+(r/n))^{nx}$ → $y = a\left(1+\left(\frac{r}{n}\right)\right)^{nx}$

where a = intial amount, r = interest rate as a decimal, n = number of times compounded per year, and x = amount of time

Annually = 1 Quarterly = 4 Daily= 365

Semi-Annually = 2 Weekly = 52

$a = 5000$

$r = 3.1\% \rightarrow .031$

$n = 4$

$x = 15$ yrs.

$y = 5000\left(1+\left(\frac{.031}{4}\right)\right)^{60}$ ($4 \cdot 15$)

$y = \$7945.81$

Answer: $7945.81

Half-Life Scenario: Actinium-226 has a half-life of 29 hours. If 100 mg of Actinium-226 disintegrates of a period of 72.5 hours, how many milligrams will remain?

$b = \frac{1}{2}$ or .5

*NOTE: Half-Life is a special DECAY scenario where your common ratio is ½ (because there is ½ remaining). X represents the NUMBER OF HALF-LIFE TIME PERIODS. Be careful with this!

$x = \frac{\text{period}}{\text{half-life}} = \frac{72.5}{29} = 2.5$

$y = a(.5)^x$

$y = 100(.5)^{2.5}$

$a = 100$

Answer: 17.68 mg