Math 3 — Name: Key

Unit 2 Day 3 Notes – Intro to Exponential and Logarithmic Functions — Date:

1. **Exponential Function:** A function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

   **Exponential Functions are functions whose equations contain a variable in the exponent!!

   Exponential Functions have the following characteristics:

   - The functions is continuous and one-to-one
   - The domain is the set of all real numbers
   - The x-axis is an asymptote of the graph.
   - The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
   - The graph contains the point $(0, a)$. That is the y-intercept is a.
   - The graphs of $y = ab^x$ and $y = a(\frac{1}{b})^x$ are reflections across the y-axis.

   Examples:

   $f(x) = 2^x$

   $g(x) = 10^x$

   $h(x) = 3^{x+1}$

   NOT Examples:

   $f(x) = x^2$

   $g(x) = 1^x$

   $h(x) = x^x$

2. **Logarithmic Function:** The function $x = log_b\, y$, where $b > 0$ and $b \neq 1$, is called a logarithmic function. This function is the inverse of the exponential function $y = b^x$ and has the following characteristics:

   - The function is continuous and one-to-one.
   - The domain is the set of all positive real numbers.
   - The y-axis is an asymptote of the graph.
   - The range is the set of all real numbers.
   - The graph contains the point $(1, 0)$. That is the x-intercept is 1.

Logarithm: In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the ***logarithm*** of x. It is usually written as $y = log_b\, x$ and is read "*y equals log base b of x.*"

**The inverse function of the exponential functions with base b, is called the logarithmic function with base b.

For $x > 0, b > 0, b \neq 0$,

$$b^x = y \quad \Longrightarrow \quad x = \log_b y$$

EXPONENTIAL FORM — LOGARITHM FORM

3. **Rewriting in both forms.**

Example 1) Rewrite logarithmic each equation in its equivalent exponential form.

a. $\log_5 x = 2$ $5^2 = x$

d. $3 = \log_b 64$ $b^3 = 64$

b. $\log_{\boxed{3}} 7 = y$ $3^y = 7$

e. $3 = \log_7 x$ $7^3 = x$

c. $2 = \log_{\boxed{b}} 25$ $b^2 = 25$

f. $\log_4 26 = y$ $4^y = 26$

Example 2) Rewrite each exponential equation in its equivalent logarithmic form.

a. $12^2 = x$ $\log_{12} x = 2$

d. $b^3 = 8$ $\log_b 8 = 3$

b. $2^5 = x$ $\log_2 x = 5$

e. $b^3 = 27$ $\log_b 27 = 3$

c. $8^3 = c$ $\log_8 c = 3$

f. $4^y = 9$ $\log_4 9 = y$

4. **Basic and Inverse Log Properties-** Because logs are exponents, they have properties that can be verified using the properties of exponents.

Basic Properties:

1. $\log_b b = 1$ because $b^1 = b$
2. $\log_b 1 = 0$ because $b^0 = 1$

Inverse Properties: (Cancel with the same base!)

1. $\cancel{\log_b b}^{\boxed{x}} = x$
2. $\cancel{b}^{\log_b \boxed{x}} = x$

Example 3) Evaluate using the log properties.

a. $\log_7 7 = 1$

e. $\log_9 9 = 1$

b. $\log_5 1 = 0$

f. $\log_8 1 = 0$

c. $\log_4 4^{\boxed{5}} = 5$

g. $6^{\log_6 9} = 9$

d. $\log_7 7^8 = 8$

h. $3^{\log_3 17} = 17$

5. **Common Logarithm:** Base 10 Logarithm, usually written **without the subscript 10.**
$\log_{10} x = \log x$, $x > 0$. Most calculators have a $\boxed{\text{LOG}}$ key for evaluating common logarithms.
The calculator is programmed in base 10.

4 decimal places

Example 4) Find the value of each log. Round to the nearest ten-thousandths.

a. $\log 81{,}000$

≈ 4.9085

c. $\log 0.35$

≈ -0.4559

b. $\log 6$

≈ 0.7782

d. $\log 0.0027$

≈ -2.5686

6. **Evaluating Logs using the Change of Base Formula**

For all positive numbers, a, b, and n, where $a \neq 1$ *and* $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Example: $\log_5 12 = \frac{\log_{10} 12}{\log_{10} 5}$

This formula allows us to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

Example 5) Evaluate each logarithm

a. $\log_3 18 = \frac{\log(18)}{\log(3)} \approx 2.6309$

b. $\log_4 25 = \frac{\log 25}{\log 4} \approx 2.3219$

c. $\log_2 16 = \frac{\log 16}{\log 2} = 4$

d. $\log_{25} 5 = \frac{\log 5}{\log 25} = \frac{1}{2}$

e. $\log_2 1024 = \frac{\log 1024}{\log 2} = 10$

f. $\log_5 125 = \frac{\log 125}{\log 5} = 3$

7. **Solving for variables with exponentials and logs.**

****MAY HAVE TO REWRITE AND APPLY PROPERTIES OR CHANGE OF BASE FORMULA!!!

Example 6) Solve for the variable:

Change of Base

a. $\log_3 \boxed{243} = y$

$\frac{\log 243}{\log 3} = y$

$\underline{y = 5}$

Change to exponential form

b. $\log_9 x = -3$

$9^{-3} = x$

$\frac{1}{9^3} = x$

$\boxed{\frac{1}{729} = x}$

c. $\log_8 n = \frac{4}{3}$

$8^{\left(\frac{4}{3}\right)} = n$

$n = 16$

Example 7) Evaluate:

a. $\log_8 8^{\boxed{4}} = x$

$x = 4$

b. $\log_9 9^{\boxed{2}} = y$

$\underline{y = 2}$

Example 8) Solve each log equation. Be sure to check your answers!

a. $\log_3(3x-6) = \log_3(2x+1)$

b. $\log_6(3x-1) = \log_6(2x+4)$

c. $\log_8(x^2-14) = \log_8(5x)$

d. $\log_4 x^2 = \log_4(4x-3)$

e. $\log_5(x-7) = 2$

f. $\log_2(4x+1) = 5$