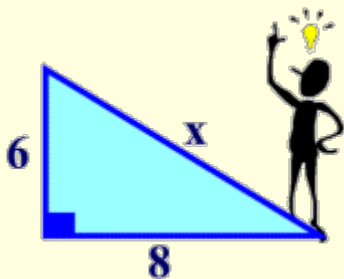


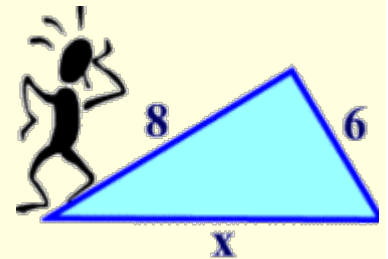
**Introduction:**

When you see the triangle below on the left and someone asks you to find the value of  $x$ , you immediately know how to proceed. You call upon your friend the Pythagorean Theorem because you see the right angle.

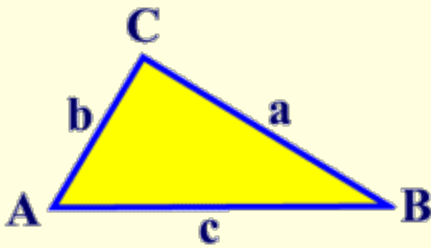


But what do you do when you see the triangle on the right? There is no indication of a right angle.

Now, with our knowledge of trigonometry, we are armed to attack any of these perplexing problems!



Let's see how to apply trigonometry to working in triangles which do not contain a right angle.



In this diagram, notice how the triangle is labeled. The capital letters for the vertices are repeated in small case on the side opposite the corresponding vertex.

side a is opposite  $\angle A$   
side b is opposite  $\angle B$   
side c is opposite  $\angle C$

working together as partners!

The ratios of each side to the sine of its "partner" are equal to each other.

That is, the **Law of Sines** states that in ANY triangle (not just right triangles) the lengths of the sides are proportional to the sines of the corresponding opposite angles.

**Formula:**

$$\frac{\text{Big Letters (Angles)}}{\text{Little Letters (sides)}} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- All are equal! These ratios, in pairs, are applied to solving problems. You never need to use all three ratios at the same time. Mix and match the ratios to correspond with the letters you need. Remember when working with proportions, cross multiply to solve.

We use the law of sines if you are given a triangle with:

**Angle-Angle-Side (AAS)** – two angles and one adjacent side

**Angle-Side-Angle (ASA)** – two angles and their included side

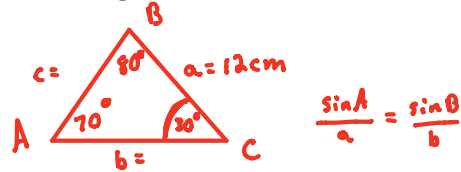
**Side-Side-Angle (SSA)** - Ambiguous case

**Examples**

1. **AAS (angle-angle-side):**

You are given a triangle, ABC, with angle A = 70°, angle B = 80° and side a = 12 cm. Find the measures of angle C and sides b and c.

- a. Draw your triangle and label the given information. (Remember, in this section, angles are named with capital letters and the side opposite an angle is named with the same lower case letter. See triangle ABC on previous page)



- b. Find Angle C by subtracting the two known angles from 180.

$m\angle C = 30^\circ$

- c. Solve for side b by setting up a proportion using the Law of Sines

~~$\frac{\sin 70}{12} = \frac{\sin 80}{b}$~~        $\frac{b \sin 70}{\sin 70} = \frac{12 \sin 80}{\sin 70}$

Cross multiply to solve for b.

$b = 12.58 \text{ cm}$

- d. Solve for side c by setting up a proportion using the Law of Sines

~~$\frac{\sin 70}{12} = \frac{\sin 30}{c}$~~        $\frac{c \sin 70}{\sin 70} = \frac{12 \sin 30}{\sin 70}$

Cross multiply to solve for c.

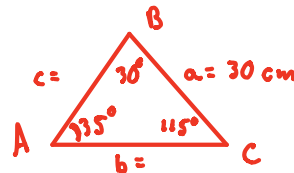
$c = 6.39 \text{ cm}$

(You should have  $m\angle C = 30$ ,  $b = 12.6$ ,  $c = 6.4$ )

2. **ASA (Angle-side-angle):**

You are given a triangle, ABC, with angle C = 115°, angle B = 30° and side a = 30 cm. Find the measures of angle A and sides b and c.

- a. Draw your triangle and label the given information.



- b. Find  $m\angle A = 35^\circ$  (Note: We MUST find angle A first because the only side given is side a.)

- c. Solve for side b by setting up a proportion using the Law of Sines

~~$\frac{\sin 35}{30} = \frac{\sin 30}{b}$~~        ~~$\frac{b \sin 35}{\sin 35} = \frac{30 \sin 30}{\sin 35}$~~        $b \approx 26.15 \text{ cm}$

- d. Solve for side c by setting up a proportion using the Law of Sines

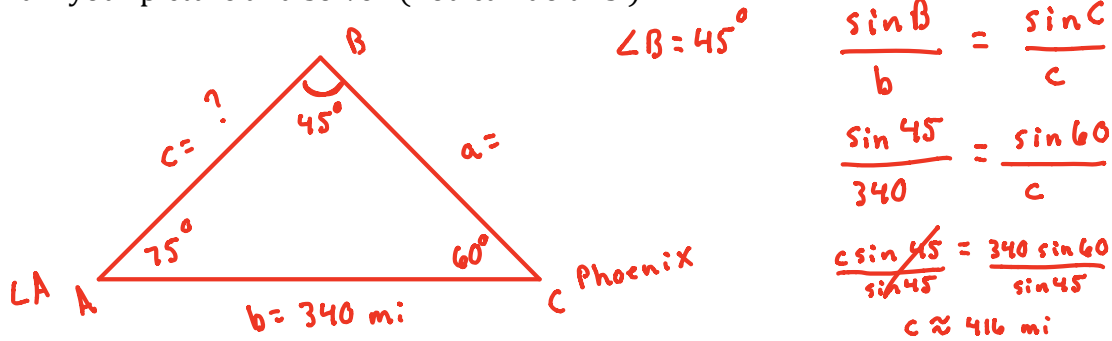
~~$\frac{\sin 35}{30} = \frac{\sin 115}{c}$~~        ~~$\frac{c \sin 35}{\sin 35} = \frac{30 \sin 115}{\sin 35}$~~        $c \approx 47.4 \text{ cm}$

Example 2 (solution)

- You should have gotten:
- Angle A = 35°**
  - Side b = 26.2 cm**
  - Side c = 47.4 cm**

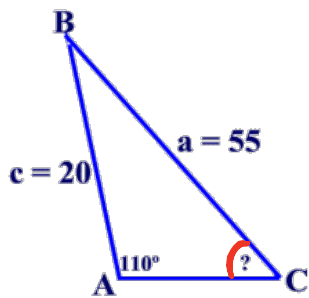
3. A satellite orbiting the earth passes directly overhead at observation stations in Phoenix and Los Angeles, 340 miles apart. At an instant when the satellite is between these two stations, its angle of elevation is simultaneously observed to be 60° at Phoenix and 75° at Los Angeles. How far is the satellite from Los Angeles?

Draw your picture and solve! (You can do this!)



(Answer should be 416.4 miles – Make sure your work justifies this answer and you know how to do this problem!)

4. You may also use Law of Sines to solve for a missing angle.  
 $a = 55$ ,  $c = 20$ , and  $m\angle A = 110^\circ$ . Find the measure of  $\angle C$  to the nearest degree.



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 110}{55} = \frac{\sin C}{20}$$

$$55 \sin C = 20(\sin 110^\circ)$$

$$\sin C = \frac{20(\sin 110^\circ)}{55}$$

$$\sin C = \left( \frac{20(0.940)}{55} \right)$$

$$\sin^{-1} C = (.342)$$

\*\*This is not your final answer. Use  $\sin^{-1}$  to solve for C!

$$m\angle C = \underline{20^\circ}$$

We will work more with SSA situations tomorrow. This can lead to an ambiguous situation where we our solution may not work or we may have more than one solution.

In Math 2, you found that we could prove two triangles congruent using:



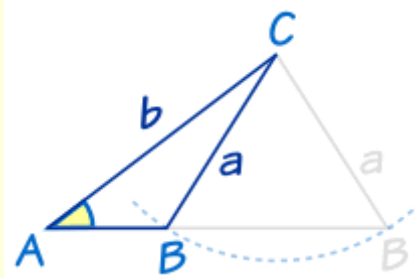
SAS - Side, Angle, Side  
 ASA - Angle, Side, Angle  
 AAS - Angle, Angle, Side  
 SSS - Side, Side, Side  
 HL - Hypotenuse Leg for Right Triangles.

We also discovered that SSA did not work to prove triangles congruent.

By definition, the word **ambiguous** means *open to two or more interpretations*.  
 Such is the case for certain solutions when working with the Law of Sines.

- If you are given two angles and one side (ASA or AAS), the Law of Sines will nicely provide you with ONE solution for a missing side.
- Unfortunately, the **Law of Sines has a problem dealing with SSA**. If you are given two sides and one angle (where you must *find an angle*), the Law of Sines could possibly provide you with one or more solutions, or even no solution.

Problem when finding angle.



Before we investigate this situation, there are a few facts we need to remember.

#### Facts we need to remember:

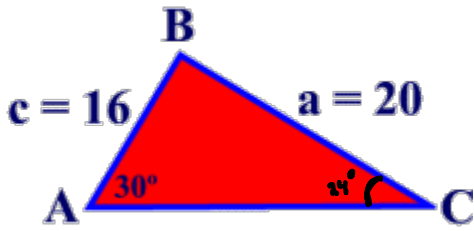
1. In a triangle, the sum of the interior angles is  $180^\circ$ .
2. No triangles can have two obtuse angles.
3. The sine function has a range of  $-1 \leq \sin \theta \leq 1$ .
4. If the  $\sin \theta =$  positive decimal  $< 1$ , the  $\theta$  can lie in the first quadrant (acute  $<$ ) or in the second quadrant (obtuse  $<$ ).

Remember!

What is the  $\sin 30^\circ$ ? .5; What is the  $\sin 150^\circ$ ? .5

Let's look at some cases. In each example, decide whether the given information points to the existence of one triangle, two triangles or no triangles.

**Example 1:** In  $\triangle ABC$ ,  $a = 20$ ,  $c = 16$ , and  $m\angle A = 30^\circ$ . How many distinct triangles can be drawn given these measurements?



Use the Law of Sines to solve for angle C:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 30}{20} = \frac{\sin C}{16}$$

$$\frac{20 \sin C}{20} = \frac{16 \sin 30}{20}$$

$$\sin^{-1} = .4 < 1$$

$$C = 24^\circ$$

1  $\triangle$

$$A = 30^\circ$$

$$B = 126^\circ$$

$$c = 24^\circ$$

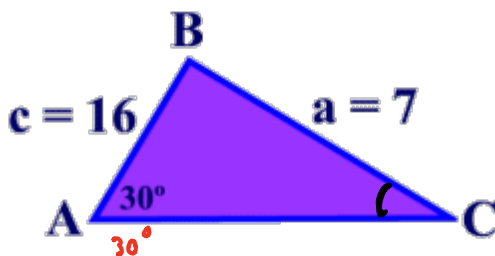
$$180 - 24 = 156^\circ$$

(potential  
 $\angle$  for C)

$$156 + 30 = 186^\circ > 180$$

1 possible  $\triangle$

**Example 2:** In  $\triangle ABC$ ,  $a = 7$ ,  $c = 16$ , and  $m\angle A = 30^\circ$ . How many distinct triangles can be drawn given these measurements?



Use the Law of Sines:

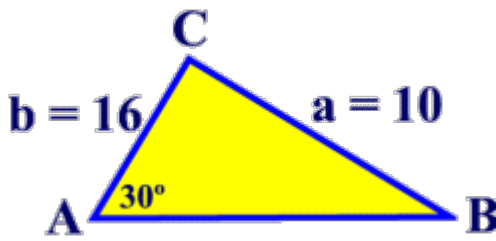
$$\frac{\sin 30}{7} = \frac{\sin C}{16}$$

$$\frac{7 \sin C}{7} = \frac{16 \sin 30}{7}$$

$$\sin^{-1} = 1.14 > 1$$

No possible  $\triangle$ 's

**Example 3:** In  $\triangle ABC$ ,  $a = 10$ ,  $b = 16$ , and  $m\angle A = 30^\circ$ . How many distinct triangles can be drawn given these measurements?



$$180 - 30 = 150 < 180$$

$$127 + 30 = 157 < 180$$

2 possible  $\Delta$ 's

Use the Law of Sines:

$$\frac{\sin 30}{10} = \frac{\sin B}{16}$$

$$\frac{10 \sin B}{10} = \frac{16 \sin 30}{10}$$

$$\sin^{-1} = .8 < 1$$

$$B = 53^\circ$$

1st	2nd
$A = 30^\circ$	$A = 30^\circ$
$B = 53^\circ$	$B = 127^\circ$
$C = 97^\circ$	$C = 23^\circ$

This example is the **Ambiguous Case**. The information given is the postulate SSA, but the two triangles that were created are clearly not congruent. We have two triangles with two sides and the non-included angle congruent, but the triangles are not congruent to each other.

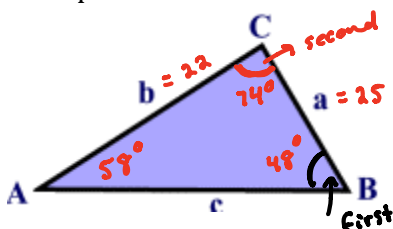
Above examples from <http://regentsprep.org/Regents/math/algtrig/ATT12/lawofsinesAmbiguous.htm>

**Summary:**

- If when using the law of sines you get an equation where  $\sin \Theta > 1$ , then there is **no solution**.
- If when using the law of sines you get an equation where  $\sin \Theta < 1$ , **solve for the angle and subtract answer from 180. Add your answer and given angle.**
  - If  $> 180$ , there is only one solution (1 triangle).
  - If  $< 180$ , there are 2 solutions (2 triangles).

**Revisiting our area of a triangle formula from Lesson 3:**

Example 6: Find the area of the triangle below.



$$m\angle A = 58^\circ$$

$$a = 25$$

$$b = 22$$

$$A = \frac{1}{2} ab \sin C \text{ (included } \angle)$$

$$= \frac{1}{2} (25)(22) \sin 74$$

$$A \approx 264$$

$$\frac{\sin 59}{25} = \frac{\sin B}{22}$$

$$\frac{25 \sin B}{25} = \frac{22 \sin 59}{25}$$

$$\sin^{-1} = .75$$

$$B = 48^\circ$$

$$180 - (59 + 48)$$

$$C = 74^\circ$$