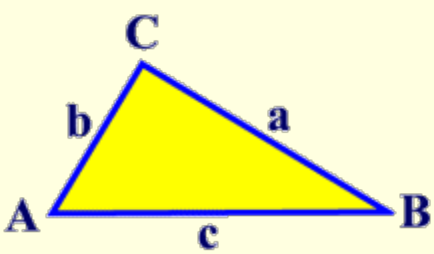
AFM

Name Key

Unit 2 Day 5 Notes – Law of Cosines

Date ______________________

In triangle problems dealing with 2 sides and 2 angles we have seen that the Law of Sines is used to find the missing item. There are many problems, however, that deal with all three sides and only one angle of the triangle. For these problems we have another method of solution called the Law of Cosines.

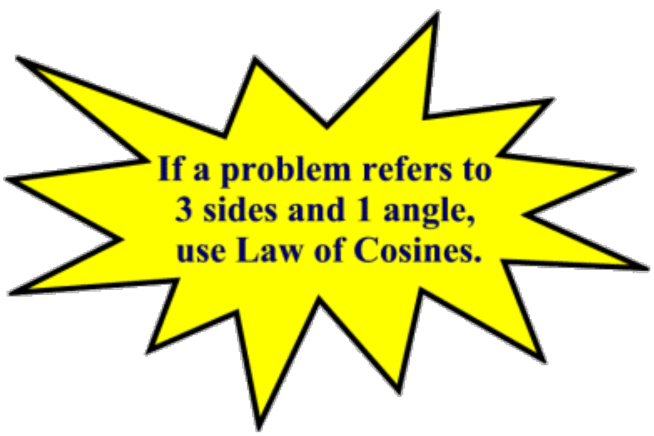


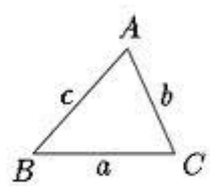
With the diagram labeled at the left, the Law of Cosines is as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Notice that $\angle C$ and side c are at opposite ends of the formula. Also, notice the resemblance (in the beginning of the formula) to the Pythagorean Theorem.

We can write the Law of Cosines for each angle around the triangle. Notice in each statement how the pattern of the letters remains the same.



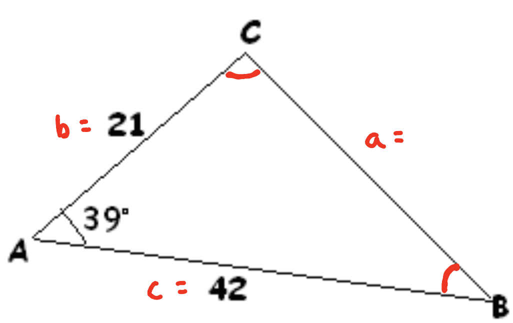


$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

SSS- Know all three sides and no angles

SAS- Know 2 sides and the angle between them

Example 1: In ΔABC, m∠A=39, AC=21 and AB=42. Find side a to the nearest integer.



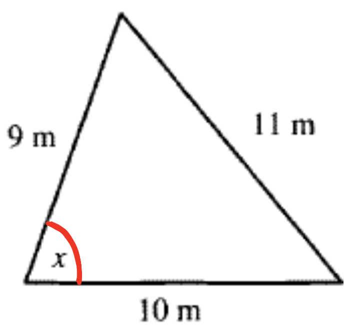
$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 21^2 + 42^2 - 2(21)(42)\cos 39$

$\sqrt{a^2} = \sqrt{834.11}$

$a \approx 28.9 \approx 29$

Example 2: In the triangle below, find the measure of angle x.



$11^2 = 9^2 + 10^2 - 2(9)(10)\cos x$

$-9^2 - 10^2 \quad -9^2 - 10^2$

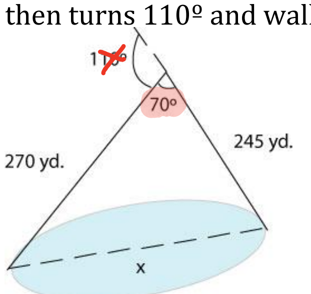
$-60 = -180\cos x$

$\frac{-60}{-180} = \frac{-180\cos x}{-180}$

$\cos^{-1} = \left(\frac{-60}{-180}\right)$

$x \approx 71^{\circ}$

Example 3: To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 yards. He then turns 110º and walks 270 yards until he arrives at the other end of the lake. Approximately how long is the

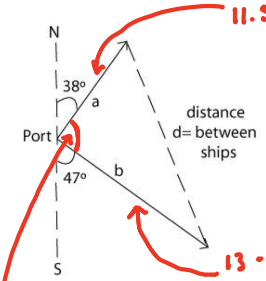


$x^2 = 270^2 + 245^2 - 2(270)(245)\cos 70$

$\sqrt{x^2} = \sqrt{87675.74}$

$x \approx 296$ yd.

Example 4: Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?



$11.5 \cdot 2 = 23$ mi

$13 \cdot 2 = 26$ mi

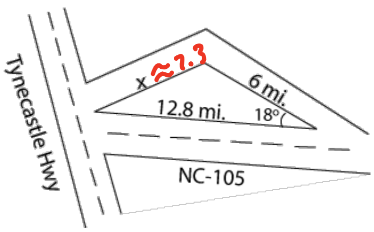
$180 - 38 - 47 = 95^{\circ}$

$d^2 = 23^2 + 26^2 - 2(23)(26)\cos 95$

$\sqrt{d^2} = \sqrt{1309.24}$

$d \approx 36$ mi

Example 5: You are heading to Beech Mountain for a ski trip. Unfortunately, state road 105 in North Carolina is blocked off due to a chemical spill. You have to get to Tynecastle Highway which leads to the resort at which you are staying. NC-105 would get you to Tynecastle Hwy in 12.8 miles. The detour begins with an 18 degree veer off onto a road that runs through the local city. After 6 miles, there is another turn that leads to Tynecastle Hwy. Assuming that both roads on the detour are straight, how many extra miles are you traveling to reach your destination?



$$x^2 = 12.8^2 + 6^2 - 2(12.8)(6)\cos 18$$

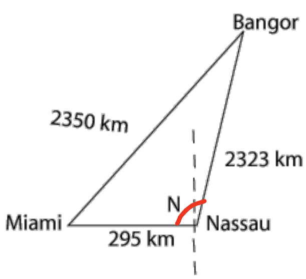
$$x^2 = 53.76$$

$$x \approx 7.3 \text{ mi}$$

Detour:
$7.3 + 6 = 13.3$ mi

$13.3 - 12.8$
$= .5$ mi
extra miles

Example 6: The distance on a map from the airport in Miami, FL to the one in Nassau, Bahamas is 295 kilometers due east. Bangor, Maine is northeast of both cities; its airport is 2350 kilometers from Miami and 2323 kilometers from Nassau. What bearing would a plane need to take to fly from Nassau to Bangor?



$$2350^2 = \cancel{295^2} + \cancel{2323^2} - 2(295)(2323)\cos x$$
$$-295^2 - 2323^2 \quad -\cancel{295^2} \quad -\cancel{2323^2}$$

$$\frac{39146}{-1370570} = \frac{-1370570\cos x}{-1370570}$$

$$\cos^{-1} = \left(\frac{39146}{-1370570}\right)$$

$$x \approx 92^\circ$$

~~**Example 7:** After the hurricane, the small tree in my neighbor's yard was leaning. To keep it from falling, we nailed a 6-foot strap into the ground 4 feet from the base of the tree. We attached the strap to the tree 3½ feet above the ground. How far from vertical was the tree leaning?~~