Math 3
Unit 7 Day 5 Notes – Solving Exp. & Log. Equations

Name Key
Date

Solving equations with NO logs!

Method 1: Similar Bases
(Note: Does not work for every problem)

Step 1: Isolate the Base

Step 2: Write both sides of the equation as an exponential with like bases.

Step 3: Set exponents equal to each other.

Step 4: Solve for the unknown.

Example 1: $2^{2x+1} = 32^x$

$2^{2x+1} = 2^{5x}$

$\cancel{2x} + 1 = 5x$
$-\cancel{2x} \quad -2x$

$\frac{1}{3} = \frac{3x}{3}$

$\boxed{x = \frac{1}{3}}$

Example 2: $\cancel{-5} + 5^{3x-9} = 120$
$+\cancel{5} \quad +5$

$5^{3x-9} = 125$

$5^{3x-9} = 5^3$

$3x \cancel{-9} = 3$
$\cancel{+9} \quad +9$

$\frac{3x}{3} = \frac{12}{3}$

$\boxed{x = 4}$

Example3: Solve for x: $3^{2x} = 27$

$3^{2x} = 3^3$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{3}{2}$

$\boxed{x = \frac{3}{2}}$

You Try! Solve for x: $2^x = 8$

$2^x = 2^3$

$\boxed{x = 3}$

Why would you need to use a log? Because the variable is in the ______________ and logs bring them down!!

Method 2: Properties of Logs

Step 1: Make sure the piece with the unknown exponent is ______________ on one side.

Step 2: ______________ the logarithm to each side.

Step 3: Use the ________________ to bring down the exponent and solve!

Example 1: Solve for x: $5^{3x} = \frac{1}{125}$

$\log 5^{3x} = \log \frac{1}{125}$

$\frac{3x \cancel{\log 5}}{\cancel{\log 5}} = \frac{\log\left(\frac{1}{125}\right)}{\log 5}$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-3}{3}$

$\boxed{x = -1}$

You Try! Solve for x: $2^{5x+1} = 32$

$\log 2^{5x+1} = \log 32$

$\frac{(5x+1) \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 32}{\log 2}$

$5x \cancel{+1} = 5$
$\quad -1$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{4}{5}$

$\boxed{x = \frac{4}{5}}$

Example 2: Solve for x: $3^x + \cancel{5} = 40$
$-\cancel{5} \quad -5$

$3^x = 35$

$\log 3^x = \log 35$

$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 35}{\log 3}$

$x \approx 3.2362$

You Try! Solve for x: $\frac{2(6^{2x})}{2} = \frac{20}{2}$

$6^{2x} = 10$

$\log 6^{2x} = \log 10$

$\frac{2x \cancel{\log 6}}{\cancel{\log 6}} = \frac{\log 10}{\log 6}$

$\frac{2x}{2} = \frac{1.2851}{2}$

$\boxed{x \approx .6425}$

The Many Ways to Solve a Logarithmic Equation

One Log	**SWOOSH!** Use when a variable is attached to the logarithm.	Rewrite into exp. form Solve for x: $\log_4(4x-2)=3$ $4^3=4x-2$ $64=4x-2$ $+2 \quad\quad +2$ $\frac{66}{4}=\frac{4x}{4}$ $x=16.5$ or $33/2$
	Change of Base Use when the variable is <u>not</u> attached to the logarithm.	Change of Base formula Solve for x: $\log_2 45 = x$ $\frac{\log(45)}{\log(2)}=x$ $\boxed{x\approx 5.4919}$
Two Logs	**Cancel the logs!** Do this if and only if there is <u>one</u> log per side.	Solve for x: $\cancel{\log_6} x = \cancel{\log_6} 2x-2$ $x=2x-2$ $-2x \quad -2x$ $-x=-2$ $\frac{-x}{-1}=\frac{-2}{-1}$ $x=2$
	Condense the logs So that only one log appears per side. Then, decide whether to cancel, swoosh, or use change of base.	1). Condense to one logarithm 2). Rewrite into exp. form Solve for x: $3\log_2 x + \log_2 5 = 7$ power 1st product $\log_2 x^3 + \log_2 5 = 7$ $\log_2 5x^3 = 7$ $2^7 = 5x^3$ $\frac{128}{5}=\frac{5x^3}{5}$ $\sqrt[3]{25.6}=\sqrt[3]{x^3}$ $\boxed{x\approx 2.9472}$
No Logs	**Add a Log!** Use this if you cannot get similar bases.	Solve for x: $7^{x-3}+5=30$ $-5 \quad -5$ $7^{x-3}=25$ $\log 7^{x-3}=\log 25$ $\frac{(x-3)\log 7}{\log 7}=\frac{\log 25}{\log 7}$ $x-3=1.6541$ $+3 \quad +3$ $\boxed{x\approx 4.6541}$
	Similar Bases! Break each base down so that they are the same, cancel the bases, and work only with the exponents!	Solve for x: $25^{2x}=125$ $5^{2(2x)}=5^3$ $2(2x)=3$ $\frac{4x}{4}=\frac{3}{4}$ $\boxed{x=\frac{3}{4} \text{ or } .75}$ $25^{2x}=125$ $\log 25^{2x}=\log 125$ $\frac{2x\log 25}{\log 25}=\frac{\log 125}{\log 25}$ $\frac{2x}{2}=\frac{1.5 \text{ or } \frac{3}{2}}{2}$ $\boxed{x=.75 \text{ or } \frac{3}{4}}$

Practice: Complete the following problems for extra practice using the above rules for solving logarithms.

1. $2\log_4 x = 12$

$\frac{2\log_4 x}{2}=\frac{12}{2}$
$\log_4 x = 6$
$4^6=x$
$\boxed{x=4096}$

$\log_4 x^2 = 12$
$4^{12}=x^2$
$\sqrt{16777216}=\sqrt{x^2}$
$\boxed{x=4096}$

2. $\log_{10} 5x - \log_{10} 7 = 2$

Quotient
$\log_{10}\frac{5x}{7}=2$
$10^2=\frac{5x}{7}$
$\frac{100}{1}=\frac{5x}{7}$
$\frac{5x}{5}=\frac{700}{5}$
$\boxed{x=140}$

3. Change of Base

$\log_5 15 = 3x$
$\frac{\log 15}{\log 5}=3x$
$\frac{1.6826}{3}=\frac{3x}{3}$
$\boxed{x\approx .5609}$

4. $4^{3x}\cdot 4^{2x}=1048576$

$4^{5x}=1048576$
$4^{5x}=4^{10}$
$\frac{5x}{5}=\frac{10}{5}$
$\boxed{x=2}$

$4^{5x}=1048576$
$\log 4^{5x}=\log 1048576$
$\frac{5x\log 4}{\log 4}=\frac{\log 1048576}{\log 4}$
$\frac{5x}{5}=\frac{10}{5}$
$\boxed{x=2}$