

**Evaluate Natural Base Expressions:**  
 "simply type "2nd e^x (insert number) enter" in your calculator"

a)  $e^{0.5} \approx 1.6487$

b)  $e^{-8} \rightarrow e^{-9} \approx 3.3546 e^{-4}$

c)  $e^2 \rightarrow c^2 \approx 7.3891$

d)  $e^{-1.3} \rightarrow e^{-1.3} \approx .2725$

**Natural base exponential function:**  
 $y = e^x$

**Natural Logarithm:**  
 logarithm with base e  
 $\log_e$

**Natural logarithm function:**  
 $y = \ln x$

**Evaluate Natural Base Expressions:**  
 "simply type "ln (insert number) enter" in your calculator"

a)  $\ln 3 \approx 1.0986$

b)  $\ln \frac{1}{4} \approx -1.3863$

c)  $\ln 4 \approx 1.3863$

d)  $\ln 0.05 \approx -2.9957$

**Example 3: Write each expression as a single natural logarithm. Use the properties of logs to condense!**

a)  $3 \ln 5 = \ln 5^3 = \ln 125 \approx 4.8283$   
 (power)

**Example 1: Simplify the expression**

a)  $\frac{\ln e^4}{8} = \frac{4}{8} = \frac{1}{2}$  or 0.5  
 (log base e of e = x)

b)  $\ln e^{83} = 83$

b)  $\ln 24 - \ln 6 = \ln \frac{24}{6} = \ln 4 \approx 1.3863$   
 (Quotient)

c)  $\frac{1}{3}(\ln x + \ln y) - 4 \ln z = (\ln x + \ln y)^{\frac{1}{3}} - \ln z^4$   
 (product, Quotient, power 1st)

d)  $2 \ln 8 - 3 \ln 4 = \ln 8^2 - \ln 4^3 = \ln \frac{64}{64} = \ln 1 = 0$   
 (power 1st, Quotient)

**Example 2: Simplify the expression**

a)  $10 \ln e = 10$

b)  $\ln 1 = 1 \ln e = 0$

**Example 4: Solve Base e Equations**  
 After isolating the e, use ln on each side to cancel out the e

a)  $e^4 + 3 = 9$   
 $e^4 = 6$   
 $\ln e^4 = \ln 6$   
 $4 = \ln 6$   
 $x \approx 7.9167$

b)  $5e^{-x} - 7 = 2$   
 $5e^{-x} = 9$   
 $e^{-x} = 1.8$   
 $\ln e^{-x} = \ln 1.8$   
 $-x = -0.5597$   
 $x \approx 0.5597$

c)  $3e^{-2x} + 4 = 10$   
 $3e^{-2x} = 6$   
 $e^{-2x} = 2$   
 $\ln e^{-2x} = \ln 2$   
 $-2x = 0.6931$   
 $x \approx -0.3465$

d)  $e^{3x+1} = e^{13}$   
 $3x+1 = 13$   
 $3x = 12$   
 $x = 4$

**Example 5: Solve Natural Log Equations**  
 After isolating the ln, use e on each side to cancel out the ln

a)  $\ln 5 - \ln x = 4$   
 Condense  
 $\ln \frac{5}{x} = 4$   
 $\frac{5}{x} = e^4 = 54.5981$   
 $x \approx 0.0915$

b)  $\ln(2m+3) = 8$   
 $2m+3 = e^8 = 2980.9579$   
 $2m = 2977.9579$   
 $m \approx 1488.9789$

c)  $\ln \frac{x-3}{4} = 8$   
 $\frac{x-3}{4} = e^8 = 2980.9579$   
 $x-3 = 11923.8319$   
 $x \approx 11926.8319$

d)  $3 \ln 3x^2 = 1$   
 $\ln 3x^2 = \frac{1}{3}$   
 $3x^2 = e^{\frac{1}{3}} = 1.3956$   
 $x^2 = \sqrt{\frac{1.3956}{3}} = \sqrt{0.4652}$   
 $x \approx 0.6825$

## Applications of Natural Logs and Base e

\*\*\*To calculate continuously compounded interest, we use the formula:

$$Y = Pe^{rt}$$

$Y =$  \$ invested plus interest       $r =$  rate

$P =$  \$ invested       $t =$  time

**Example 6:** How much money will be in a bank account after 1.5 years if you invested \$400 at 7.6% compounded continuously?

$$\begin{aligned}
 P &= \$400 & Y &= 400e^{(.076 \cdot 1.5)} & Y &= Pe^{rt} \\
 r &= \frac{7.6\%}{100} = .076 & Y &\approx \$448.30 \\
 t &= 1.5
 \end{aligned}$$

**Practice:** Complete the following problems for class work. Show all work.

1. Solve  $\ln(14x - 3) = \ln(7x + 11)$

$$x = 2$$

2. Solve  $2e^x - 5 = 1$

$$x \approx 1.0186$$

3.  $\ln(x - 1) = -2$

$$x \approx 1.1353$$

4.  $\ln(2x - 3) = 2.5$

$$x \approx 7.5912$$

5.  $\ln 48 - \ln x = \ln 4$

$$\begin{aligned}
 \cancel{\ln} \frac{48}{x} &= \cancel{\ln} 4 \\
 \frac{48}{x} &= 4
 \end{aligned}$$

$$x = 12$$

6.  $e^{3x} \cdot e^x = 15$

$$\begin{aligned}
 e^{3x} \cdot e^x &= 15 \\
 e^{4x} &= 15 \\
 4x &= \ln 15 \\
 x &\approx 0.6770
 \end{aligned}$$

**Mixed Review:** Remember, all logarithms share the same rules. Always condense first before solving!

7.  $4^{3x} = 12$

8.  $\log_6 x + \log_6 9 = \log_6 54$

9.  $\log_2 x = -3$

10.  $\log_2 64 = x$

11.  $\log_2 x - \log_2 5 = 3$

12.  $\ln 4x + \ln 5 = \ln 20$

13. Mazie invested \$4500 in an account earning 4.3% interest compounded continuously. After how many years will she have \$7400 in her account?