Unit 2A Day 2 Notes - Transformations w/ Fred Cont.
Date: $\qquad$
Today we will revisit Fred, our "parent" function, and investigate transformations other than translations.
Recall that the equation for Fred is $\mathbf{y}=\mathrm{F}(\mathrm{x})$.
Complete the chart with Fred's characteristic points.

| $x$ | $F(x)$ |
| :---: | :---: |
| -1 | 1 |
| 1 | -1 |
| 2 | -1 |
| 4 | -2 |


I. Let's suppose that Freddie Jr. is $\mathbf{y}=-\mathrm{F}(\mathrm{x})$

1. Complete the table.


| $y=-F(x)$ |  |  |
| :---: | :---: | :---: |
| X | $\mathrm{F}(\mathrm{x})$ | y |
| -1 | 1 | -1 |
| 1 | -1 | 1 |
| 2 | -1 | 1 |
| 4 | -2 | 2 |

2. On the coordinate plane above, graph the 4 ordered pairs ( $\mathrm{x}, \mathrm{y}$ ). [Hint: The $1^{\text {st }}$ point should be ( $-1,-$ 1).]
3. What type of transformation maps Fred, $\mathrm{F}(\mathrm{x})$, to Freddie Jr., $-\mathrm{F}(\mathrm{x})$ ? (Be specific.)

$$
\text { reflects over the } x \text {-axis }
$$

4. How did this transformation affect the x -values? (Hint: Compare the characteristic points of Fred and Freddie Jr.) $x$-values did not change
5. How did this transformation affect the y-values? (Hint: Compare the characteristic points of Fred and Freddie Jr.)

$$
y \text {-values charyed signs }
$$

6. In $\mathrm{y}=-\mathrm{F}(\mathrm{x})$, how did the negative coefficient of " $\mathrm{F}(\mathrm{x})$ " affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

$$
\text { reflected over the } x \text {-axis }
$$

II. Now let's suppose that Freddie Jr. is $\mathbf{y}=\mathrm{F}(-\mathrm{x})$

1. Complete the table.
$\mathrm{y}=\mathrm{F}(-\mathrm{x})$

| X | -x | y |
| :---: | :---: | :---: |
| 1 | -1 | 1 |
| -1 | 1 | -1 |
| -2 | 2 | -1 |
| -4 | 4 | -2 |


2. On the coordinate plane above, graph the 4 ordered pairs ( $\mathrm{x}, \mathrm{y}$ ). [Hint: The $1^{\text {st }}$ point should be (1, 1).]
3. What type of transformation maps Fred, $\mathrm{F}(\mathrm{x})$, to Freddie Jr., $\mathrm{F}(-\mathrm{x})$ ? (Be specific.)

$$
\text { reflects over the } y \text {-axis }
$$

4. How did this transformation affect the x -values? (Hint: Compare the characteristic points of Fred and Freddie Jr.) $\quad x$-values chearyed signs
5. How did this transformation affect the $y$-values? (Hint: Compare the characteristic points of Fred and Freddie Jr.) $\quad y$-value did not change
6. In $y=F(-x)$, how did the negative coefficient of " $x$ " affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

$$
\text { reflected over the } y \text {-axis }
$$

III. Checkpoint: Harry is $\mathrm{H}(\mathrm{x})$ and is shown on each grid. Use Harry's characteristic points to graph Harry's children without making a table.

1. $y=H(-x)$

2. $y=-H(x)$

IV. Now let's return to Fred, whose equation is $\mathbf{y}=\mathrm{F}(\mathrm{x})$.

Complete the chart with Fred's characteristic points.

| $x$ | $F(x)$ |
| :---: | :---: |
| -1 | 1 |
| 1 | -1 |
| 2 | -1 |
| 4 | -2 |

Let's suppose that Freddie Jr. is $y=4 F(x)$


1. Complete the table.


2. On the coordinate plane above, graph the 4 ordered pairs ( $\mathrm{x}, \mathrm{y}$ ). [Hint: The $1^{\text {st }}$ one should be $(-1,4)$.]
3. How did this transformation affect the x -values? (Hint: Compare the characteristic points of Fred and Freddie Jr.)

$$
x \text {-values did not charge }
$$

4. How did this transformation affect the y-values? (Hint: Compare the characteristic points of Fred and Freddie Jr.)

$$
y \text {-values were multiplied by } 4
$$

5. In $\mathrm{y}=4 \mathrm{~F}(\mathrm{x})$, the coefficient of " $\mathrm{F}(\mathrm{x})$ " is 4 . How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain.
vertically stretched by a factor of 4
Dilation
V. Checkpoint:
6. Complete each chart below. Each chart starts with the characteristic points of Fred.

| $\mathbf{x}$ | $\mathbf{F}(\mathbf{x})$ | $\mathbf{3 F}(\mathbf{x})$ |
| :---: | :---: | :---: |
| -1 | 1 | 3 |
| 1 | -1 | -3 |
| 2 | -1 | -3 |
| 4 | -2 | -6 |


| $\mathbf{x}$ | $\mathrm{F}(\mathrm{x})$ | $1 / 4 \mathrm{~F}(\mathrm{x})$ |
| :---: | :---: | :---: |
| -1 | 1 | $1 / 4$ |
| 1 | -1 | $-1 / 4$ |
| 2 | -1 | $-1 / 4$ |
| 4 | -2 | $-1 / 2$ |

2. Compare the $2^{\text {nd }}$ and $3^{\text {rd }}$ columns of each chart above. The $2^{\text {nd }}$ column is the $y$-value for Fred. Can you make a conjecture about how a coefficient changes the parent graph?

$$
\begin{aligned}
& \#>1 \rightarrow \text { stretches (got bigger) } \\
& 0<\#<1 \rightarrow \text { shrinks (got smaller) }
\end{aligned}
$$

VI. Now let's suppose that Freddie Jr. is $\mathbf{y}=\mathbf{- 3} \mathrm{F}(\mathrm{x})$.

1. Complete the table.
$\mathrm{y}=-3 \mathrm{~F}(\mathrm{x})$

| $\mathbf{x}$ | $\mathrm{F}(\mathrm{x})$ | $\mathbf{y}$ |
| :---: | :---: | :---: |
| -1 | -3 | -3 |
| 1 | 3 | 3 |
| 2 | 3 | 3 |
| 4 | 6 | 6 |


2. On the coordinate plane above, graph the 4 ordered pairs ( $\mathrm{x}, \mathrm{y}$ ). [Hint: The $1^{\text {st }}$ one should be $(-1,-3)$ ]
3. Reread the conjecture you made in \#7 on the previous page. Does it hold true or do you need to refine it? Yes
If it does need some work, restate it more correctly here.
VII. Checkpoint: Let's revisit Harry, H(x).

1. Describe the effect on Harry's graph for each of the following.

Example: $-5 \mathrm{H}(\mathrm{x})$ $\qquad$ Each point is reflected in the x -axis and is 5 times as far from the x -axis.
a. $3 H(x)$ vertically shetches by 3
b. $-2 \mathrm{H}(\mathrm{x})$ reflected over the $x$-axis + vertically stretches by 2
c. $\frac{1}{2} \mathrm{H}(\mathrm{x})$ vertically shrinks by $\frac{1}{2}$
2. Use your answers to questions 1 and 2 to help you sketch each graph without using a table.
a. $y=3 H(x)$
b. $\quad y=-2 H(x)$




The graph of Dipper, $\mathrm{D}(\mathrm{x})$, is shown.

List the characteristic points of Dipper.
$(-5,-1)(-2,-3)(1,-1)(7,0)$

What is different about Dipper from the functions we have used so far?

Since Dipper is our original function, we will refer to him as the parent function. Using our knowledge of transformational functions, let's practice finding children of this parent.


Note: In transformational graphing where there are multiple steps, it is important to perform the translations last.
I. Example: Let's explore the steps to graph Dipper Jr, $2 \mathrm{D}(\mathrm{x}+3)+5$, without using tables.

Step 1. The transformations represented in this new function are listed below in the order they will be performed. (See note above.)

- Vertical stretch by 2 (Each point moves twice as far from the x -axis.)
- Translate left 3.
- Translate up 5.

Step 2. On the graph, put your pencil on the left-most characteristic point, $(-5,-1)$.

- Vertical stretch by 2 takes it to $(-5,-2)$. (Note that the originally, the point was 1 unit away from the x -axis. Now, the new point is 2 units away from the x -axis.)
- Starting with your pencil at $(-5,-2)$, translate this point 3 units to the left. Your pencil should now be on $(-8,-2)$.
- Starting with your pencil at $(-8,-2)$, translate this point up 5 units. Your pencil should now be on $(-8,3)$.
- Plot the point $(-8,3)$. It is recommended that you do this using a different colored pencil.

Step 3. Follow the process used in Step 2 above to perform all the transformations on the other 3 characteristic points.

Step 4. After completing Step 3, you will have all four characteristic points for Dipper Jr. Use these to complete the graph of Dipper Jr. Be sure you use a curve in the appropriate place. Dipper is not made of segments only.
II. Dipper has another child named Little Dip, - D(x) - 4

Using the process in the previous example as a guide, graph Little Dip (without using tables).

1. List the transformations needed to graph Little Dip.
(Remember, to be careful with order.)

- reflected over the $x$-axis
- translate down 4


2. Apply the transformations listed above to each of the four characteristic points.
3. Complete the graph of Little Dip using your new characteristic points from \#2.
III. Dipper has another child named Dipsy, $3 \mathrm{D}(-\mathrm{x})$

Using the process in the previous example as a guide, graph Dipsy (without using tables).

1. List the transformations needed to graph Dipsy. (Remember, to be careful with order.)

- vertical swetch by 3
- reflected over the y-axis


2. Apply the transformations listed above to each of the four characteristic points.
3. Complete the graph of Dipsy using your new characteristic points from \#2.
IV. Now that we have practiced transformational graphing with Dipper and his children, you and your partner should use the process learned from the previous three problems to complete the following.
4. Given Cardio, C(x), graph: $y=3 C(x)+5$
5. Given Garfield, $\mathrm{G}(\mathrm{x})$, graph: $\mathrm{y}=-\mathrm{G}(\mathrm{x}-3)-\mathbf{6}$
6. Given Horizon, $H(x)$, graph $y=-3 H(x)$

7. Given Batman, $B(x)$, graph: $\quad y=B(-x)+8$

8. Given Mickey, $M(x)$, graph: $\quad y=-\frac{1}{3} M(x)$

