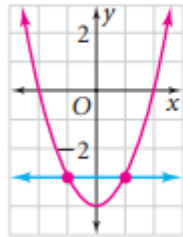


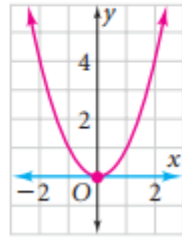
A system of equations can have zero, one or two solutions.

$y = x^2 - 4$   
 $y = -3$



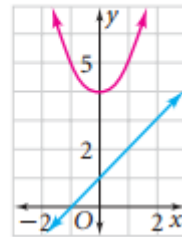
2 solutions

$y = x^2$   
 $y = 0$



1 solution

$y = x^2 + 4$   
 $y = x + 1$



No solutions

Solving a System: Two Methods

1. Graphically
2. Algebraically

Graphically:

1. Graph the two equations
2. Where the two graphs intersect, this is your solutions.

Algebraically:

1. Make sure both equation are in  $y =$  form if necessary
2. Substitute the linear equation into the 'y part' of the quadratic equation, to have only one variable left to solve in the equation.
3. Get NEW quadratic equation into standard form ( $ax^2 + bx + c$ ) and equal to zero.
4. **Since it is a quadratic:** Must **FACTOR TO SOLVE FOR X**.  
(How many answers should you get? 2)
5. Must find other variable (y) by substituting your x answers into one of the equation and solve for y.
6. Check solutions

Checking your Answer: To check on your graphing calculator (find intersection):

- 1) Go to   (calculate) and pick  (intersection)
- 2) Move cursor to wanted intersection point

Example 1: Solve the system of equations  $y = -x^2 + 4x + 1$  and  $y = -x + 5$  algebraically and by graphing

$$\begin{array}{r}
 -x^2 + 4x + 1 = -x + 5 \\
 +x \quad -5 \quad \cancel{+x} \quad \cancel{-5} \\
 \hline
 -x^2 + 5x - 4 = 0
 \end{array}$$

1st

$$\begin{array}{r}
 -1 \cdot -4 \\
 \hline
 4 \quad 1 \\
 \hline
 4 \quad 1 \\
 \hline
 5
 \end{array}$$

$$\frac{4}{-1} = -4 \quad \frac{1}{-1} = -1$$

change signs

$$\begin{array}{l}
 x = 4 \quad y = -4 + 5 = 1 \\
 x = 1 \quad y = -1 + 5 = 4
 \end{array}$$

Solutions:

$$\left\{ \begin{array}{l} (4, 1) \\ (1, 4) \end{array} \right\}$$

Example 2: Solve the system of equations algebraically and graphically.

$$\begin{array}{r}
 y = x^2 - 2 \\
 y = -x
 \end{array}$$

$$\begin{array}{r}
 x^2 - 2 = -x \\
 +x \quad +x \\
 \hline
 x^2 + x - 2 = 0
 \end{array}$$

$$\begin{array}{r}
 -2 \\
 \hline
 -1 \quad 2 \\
 \hline
 1
 \end{array}$$

change signs

Solutions:

$$\begin{array}{l}
 x = 1 \quad y = -1 \quad (1, -1) \\
 x = -2 \quad y = 2 \quad (-2, 2)
 \end{array}$$

### Applications of Systems of Equations

The student council decides to put on a concert to raise money for an after school program. They have determined that the price of the ticket will affect their profit. The functions shown below represent their potential income and cost of putting on the concert, where t represents ticket price.

$$\text{Income: } I(t) = 330t - 30t^2$$

$$\text{Cost: } C(t) = 330 - 30t$$

Using colored pencils to graph each function on the axes below and answer the questions.

- Show algebraically and graphically where the break-even point. (Hint: Income = Cost)
- Show algebraically and graphically where the cost is greater than the income.
- Show algebraically and graphically where the income is greater than the cost.
- Which ticket price would you use in order to maximize your profit? Where is this shown on the graph?

