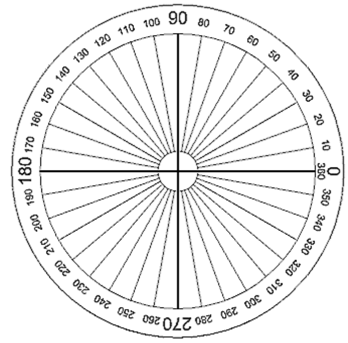
AFM

Unit 3 Day 1 Notes – Angle & Radian Measure

Name Key

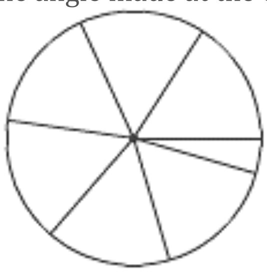
Date ______________________



<u>Units for Measuring Angles</u>

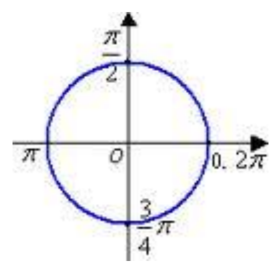
- **Degrees:** A circle is divided into 360 equal degrees, so that a right angle is $90°$

- **Radians:** One **radian** is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.



-The circumference of a circle with radius 1 is 2π so a complete revolution has made 2π radians (or approximately 6.28 radians as seen in the above figure).

-A straight angle (or half of a circle) has measure π radians.



Converting Radians and Degrees:

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees}$$

$\frac{D^{\circ}}{180}$ (reduce) $\rightarrow \frac{\#\pi}{\#}$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians}$$

$\frac{\#\cancel{\pi}}{\#} \cdot \frac{180}{\cancel{\pi}} \rightarrow D^{\circ}$

Examples:

1. Express 60º in radians

$\frac{60^{\circ}}{1} \times \frac{\pi}{180} \rightarrow \frac{60}{180} = \frac{1}{3} \rightarrow \underbrace{\frac{1\pi}{3} \text{ or } \frac{\pi}{3}}$

2. Express $\frac{\pi}{6}$ rad in degrees

$\frac{1\cancel{\pi}}{6} \times \frac{180}{\cancel{\pi}} \rightarrow \left(\frac{1}{6}\right) \times 180 \rightarrow \underline{30^{\circ}}$

On Your Own:

#1-8, change the given angle to radians.

1) 315°

$315 \times \frac{\pi}{180}$

$\frac{315}{180} = \frac{7}{4} \to \frac{7\pi}{4}$

2) -60°

$-60 \cdot \frac{\pi}{180}$

$\frac{-60}{180} = \frac{-1}{3} \to \frac{-\pi}{3}$

3) 212°

$\frac{53\pi}{45}$

4) -168°

$-\frac{14\pi}{15}$

5) 12.5°

$\frac{5\pi}{72}$

6) -310°

$-\frac{31\pi}{18}$

7) 600°

$\frac{10\pi}{3}$

8) -720°

-4π

#9-16, change the given angle to degrees.

9) $\frac{3\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}}$

$\left(\frac{3}{4}\right) \times 180 = 135°$

10) $-\frac{9\cancel{\pi}}{5} \times \frac{180}{\cancel{\pi}}$

$\left(-\frac{9}{5}\right) \times 180 = -324°$

11) $\frac{15\pi}{8}$

$337.5°$

12) $-\frac{\pi}{10}$

$-18°$

13) $\frac{7\pi}{10}$

$126°$

14) $-\frac{16\pi}{15}$

$-192°$

15) $\frac{88\pi}{9}$

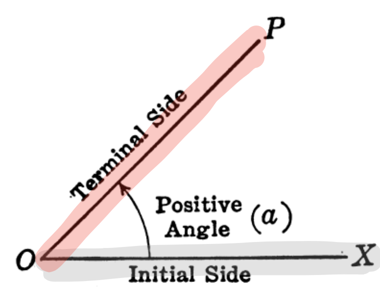
$1760°$

16) $-\frac{29\pi}{12}$

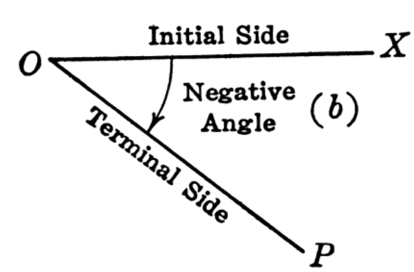
$-435°$

<u>**Angles in Standard Position**</u>

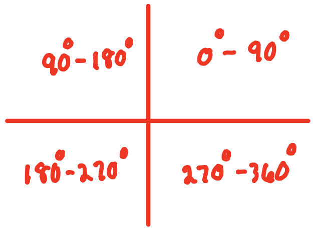
Angle: generated by the rotation of 2 rays that share a fixed endpoint

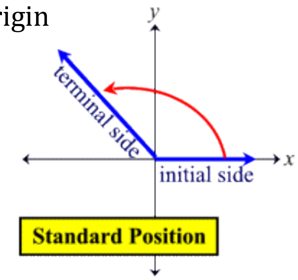


- **Initial Side:** fixed ray
- **Terminal Side:** ray that rotates away from initial side
- **Positive Angle:** counterclockwise rotation
- **Negative Angle:** clockwise rotation



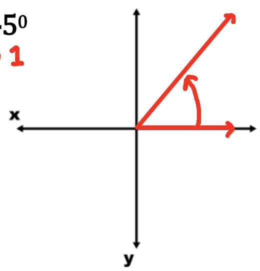
An angle is in **standard position** if it is draw in the xy-plane with its vertex at the origin and its initial side on the positive x-axis.



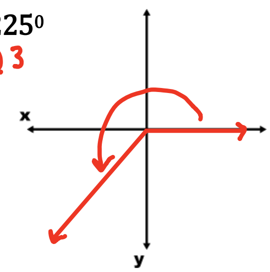


Example: Draw the given angle in standard position. State the quadrant in which the terminal side lies.

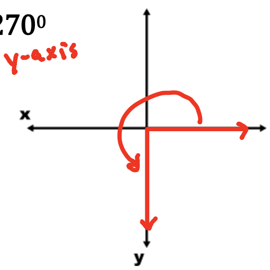
1. 450 Q1



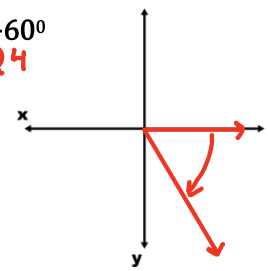
2. 225^0 Q3



3. 270^0 y-axis

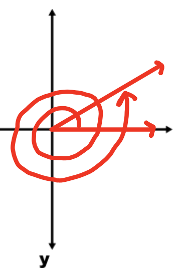


4. -60^0 Q4

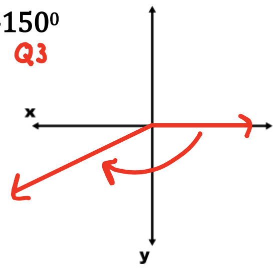


5. 750^0 Q1

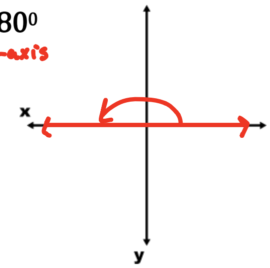
$$\begin{array}{r} 750 \\ -360 \\ \hline 390 \\ -360^\circ \\ \hline 30^\circ \end{array}$$



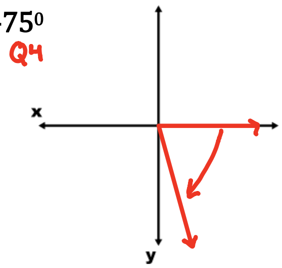
6. -150^0 Q3



7. 180^0 x-axis

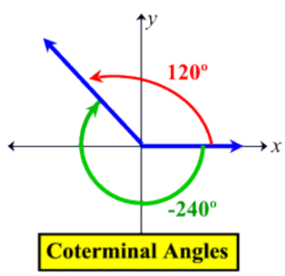


8. -75^0 Q4



Coterminal Angles

Two angles in standard position are **coterminal angles** if their terminal sides coincide. Every angle has infinitely many coterminal angles.



To find angles that are coterminal, add or subtract any multiple of 360 for degrees or 2π for radians.

Examples:

1. Find three angles that are coterminal with the angle $\theta = 30°$ in standard position

$30^\circ - 360 = -330^\circ - 360^\circ = -690^\circ - 360 = -1050^\circ$ 1). 2). 3). | $+360$ 1). $390^\circ + 360 = 750^\circ + 360 = 1110^\circ$ 2). 3).

2. Find three angles that are coterminal with the angle $\theta = \frac{\pi}{3}$ in standard position

$\frac{\pi}{3} + \frac{2\pi}{1}\frac{\times 3}{\times 3} \rightarrow \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3} + \frac{6\pi}{3} = \frac{13\pi}{3} + \frac{6\pi}{3} = \frac{19\pi}{3}$ 1). 2). 3).

3. Find an angle with a measure between 0º and 360º that is coterminal with the angle of measure 1290º in standard position.

$$\begin{array}{r} 1290^\circ \\ -360 \\ \hline 930^\circ \\ -360 \\ \hline 570 \\ -360 \\ \hline 210^\circ \end{array}$$