Math 3
Unit 3 Day 1 Notes - Intro to Polynomials

Name: Key
Date:

All Polynomials must have whole numbers as exponents!!

Example: $9x^{-1}+12x^{\frac{1}{2}}$ is NOT a polynomial.

Polynomial Vocabulary Review

- Expression: Numbers, symbols, and operators grouped together that show the value of something. $2x+3$
- Equation: An equation says that two things are equal. It will have an equal sign. $7x+2=10x+1$
- Terms: A single number or variable, or numbers and variables multiplied together; they are separated by plus or minus signs.
    - Monomial, Binomial, Trinomial, Polynomial
      one term, two terms, three terms, 4 or more terms
- Degree: The highest exponent of a polynomial expression or equation.
    - Constant, Linear, Quadratic, Cubic, Quartic, Quintic
      Degree: 0, 1, 2, 3, 4, 5

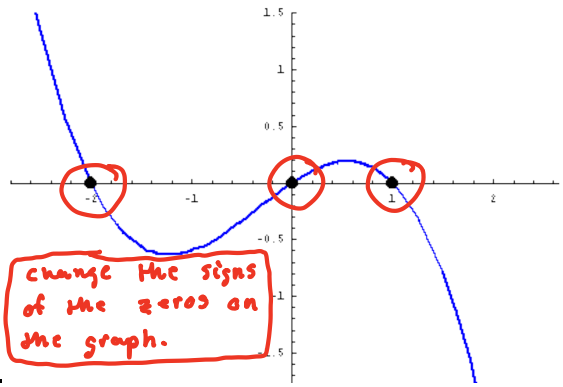
Example 1: Classifying Polynomials

POLYNOMIAL	NUMBER OF TERMS	CLASSIFICATION BY TERMS	DEGREE	CLASSFICATION BY DEGREE	SKETCH THE GRAPH
$f(x) = 5x^0$	1	monomial	0	constant	
$g(x) = 4x - 3$	2	binomial	1	Linear	
$p(x) = -2x^5$	1	monomial	5	Quintic	
$w(x) = x^4 - 4x + 2$	3	trinomial	4	Quartic	
$y = -4x^2 + x + 9$	3	Trinomial	2	Quadratic	
$h(x) = 4x^3 + x^2 - 9x + 2$	4	Polynomial	3	Cubic	

The number "k" is said to be a **zero** of a polynomial if f(k) = 0.

- "k" is often referred to as the **root** or **solution**
- If "k" is a real number, then f(k) = 0 means that the graph crosses the x-axis at that value. "k" can also be referred to as an **x-intercept**

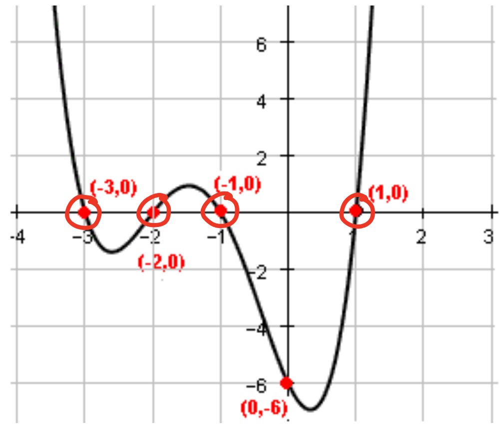
Check out the graphs below and identify any values that represent a zero/solution/root.



A.

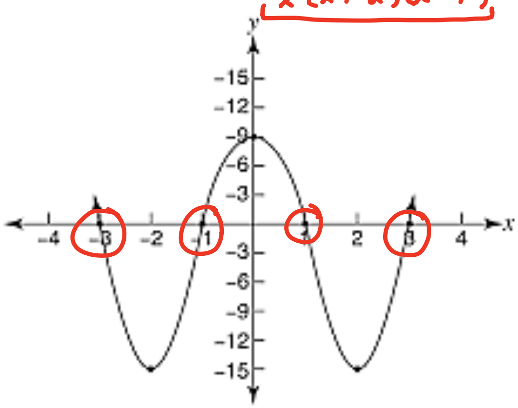
factored equation: $(x+2)(x+0)(x-1)$

$x(x+2)(x-1)$



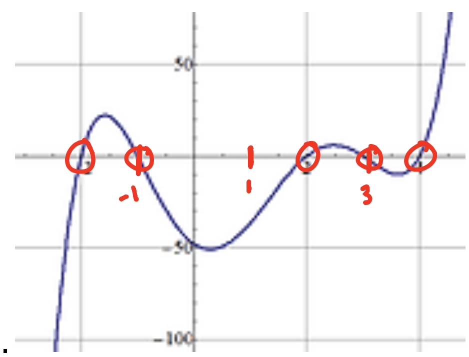
B.

factored equation: $(x+3)(x+2)(x+1)(x-1)$



C.

factored equation: $(x+3)(x+1)(x-1)(x-3)$

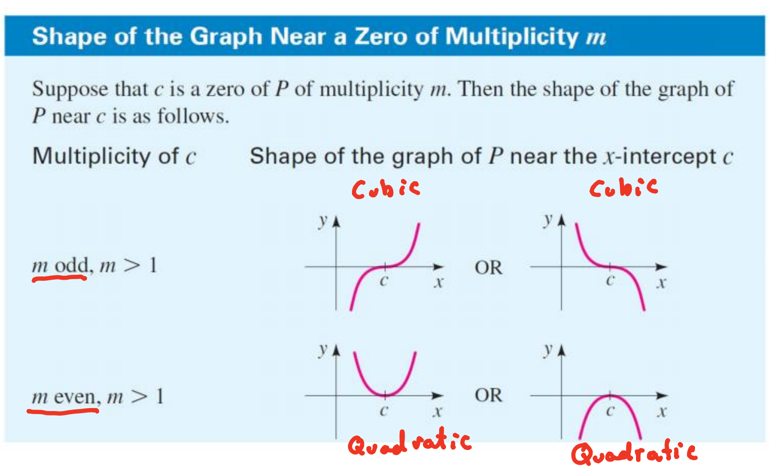


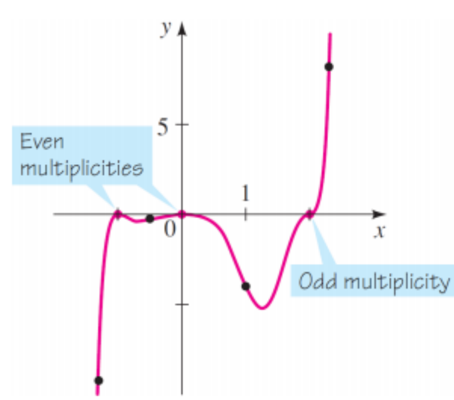
D.

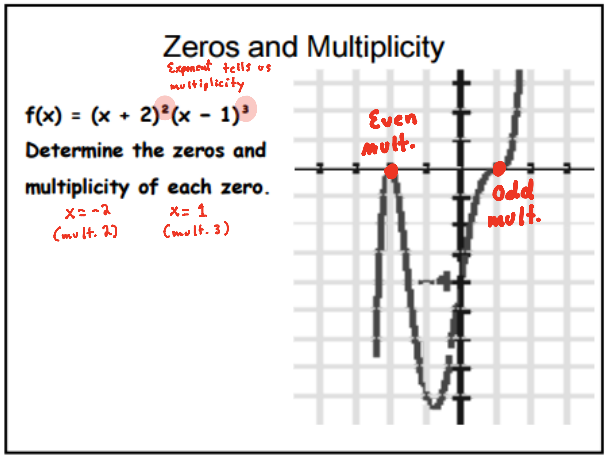
factored equation: $(x+2)(x+1)(x-2)(x-3)(x-4)$

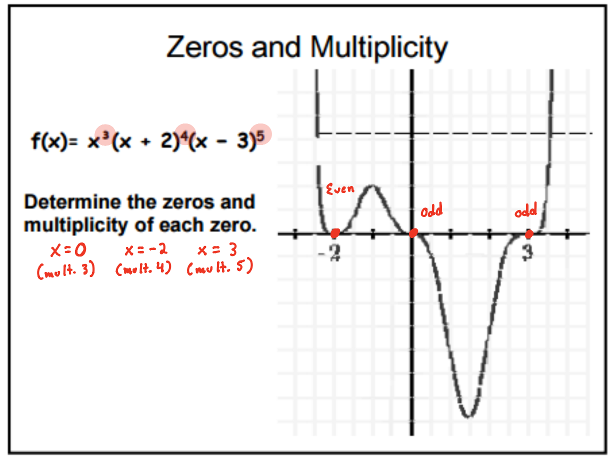
WATCH OUT! Multiplicities of Zeros

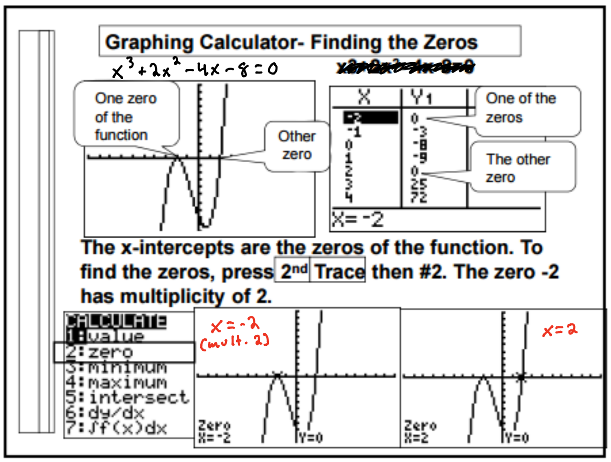
If c is a zero of the function P and the corresponding factor (x − c) occurs exactly m times in the factorization of P then we say that c is a zero of **multiplicity** m. One can show that the graph of P crosses the x-axis at c if the multiplicity m is odd and does not cross the x-axis if m is even.











Example

- Find all zeros of $f(x)$ and state the multiplicity of each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.
- $f(x) = x^3 + 5x^2 - 9x - 45$

$x = -3$ $x = -5$ $x = 3$

Example

Find the zeros of $f(x) = 3(x + 5)(x + 2)^2$ and give the multiplicity of each zero. State whether the graph crosses the x-axis or touches the x-axis and turns around at each zero.

$x = -5$ $x = -2$ (mult. 2)

The high and low points on a graph are called the **extrema** of the function. An extremum that is higher or lower than any other points nearby is called a relative extremum.

A **relative extremum** (the plural of extremum is extrema) that is higher than points nearby is called relative maximum. A relative extremum that is lower than points nearby is called a relative minimum.

A function's **absolute extremum** occurs at the highest or lowest point on a function. The highest point on a function is called the absolute maximum and the lowest point on a function is called the absolute minimum.

Can you identify these in the graphs on the previous page?

The **end behavior** of a polynomial is a description of what happens as x becomes large in the positive or negative direction. For any polynomial, the end behavior is *determined by the term that contains the highest power of x*, because when x is large, the other terms are relatively insignificant in size.

Even Polynomials

For example, the monomial $y = x^2$ has the following end behavior:

$x \to -\infty \quad y \to \infty$ $\qquad$ $x \to \infty \quad y \to \infty$

$\cancel{y \to \infty \text{ as } x \to -\infty}$ and $\cancel{y \to \infty \text{ as } x \to \infty}$

UP (left) and UP (right)

$y = -x^2$

$x \to -\infty \quad y \to -\infty$

$x \to +\infty \quad y \to -\infty$

Odd polynomials

The monomial $y = x^3$ has the following end behavior:

$x \to -\infty \quad y \to -\infty$ $\qquad$ $x \to \infty \quad y \to \infty$

$\cancel{y \to -\infty \text{ as } x \to -\infty}$ and $\cancel{y \to \infty \text{ as } x \to \infty}$

DOWN (left) and UP (right)

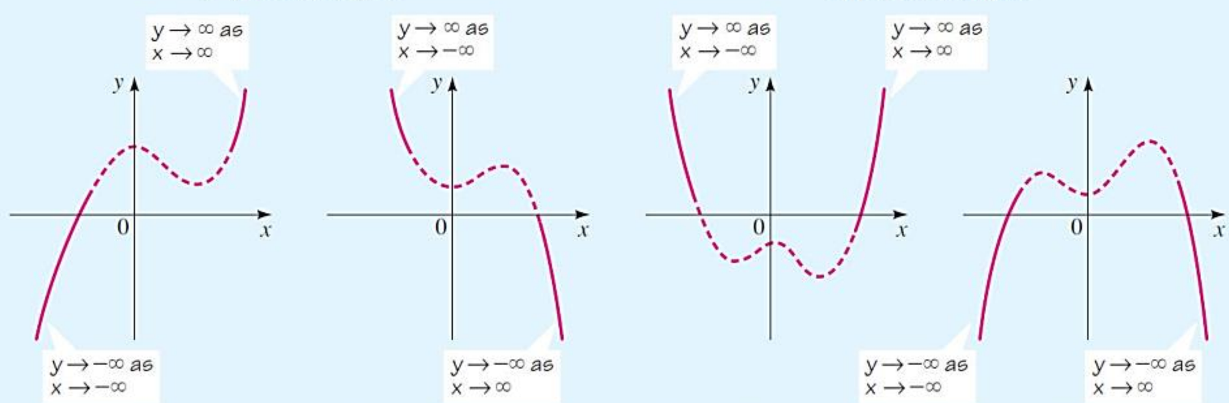
$y = -x^3$

$x \to -\infty \quad y \to \infty$

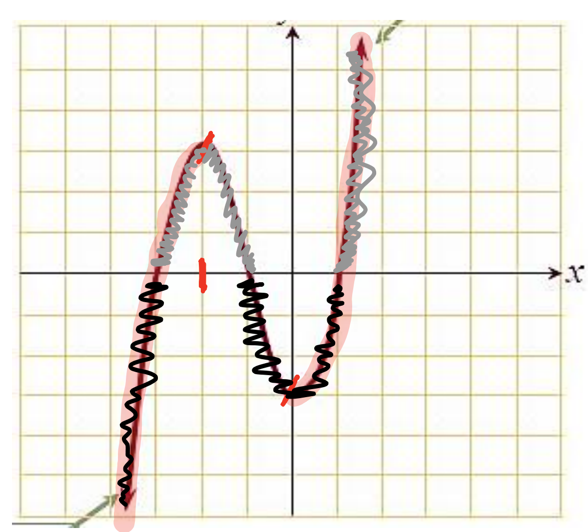
$x \to \infty \quad y \to -\infty$

End Behavior of Polynomials

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.



Given the graph of the polynomial below



- State the intervals where the graph is **increasing/decreasing** (think "slope")

  Increasing: $(-\infty, -2)$
  $(0, \infty)$

  Decreasing: $(-2, 0)$

- State the intervals where the graph is **positive/negative** (above/below the x-axis)

  Positive: $(-3, -1)$
  $(1, \infty)$

  Negative: $(-\infty, -3)$
  $(-1, 1)$