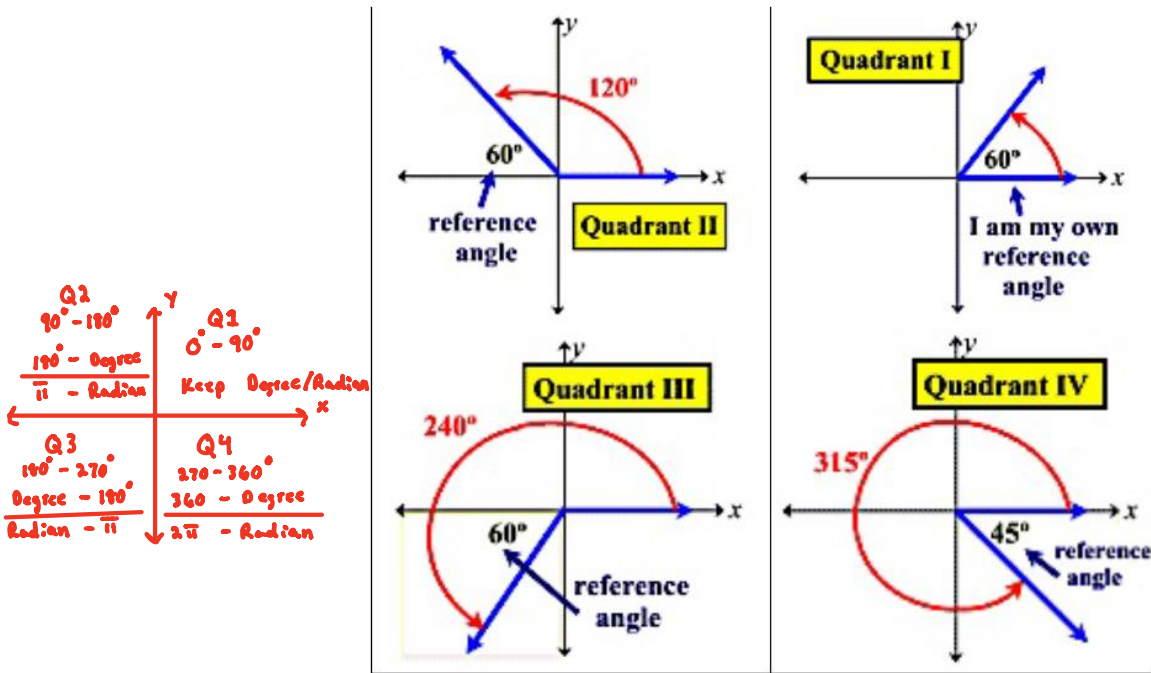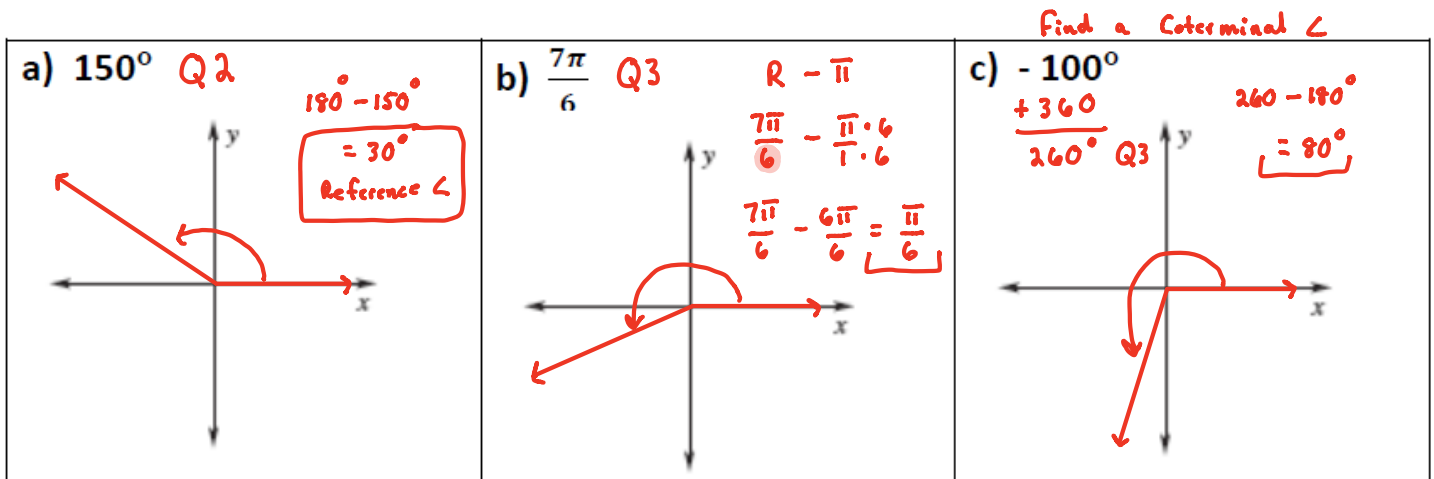An angle drawn in standard position has a **reference angle**. The reference angle is an acute angle formed by the terminal side of the given angle to the x-axis.



Q2  90°–180°
Q1  0°–90°
180 – Degree
π – Radian    Keep Degree/Radian

Q3  180°–270°
Degree – 180
Radian – π

Q4  270°–360°
360 – Degree
2π – Radian

**120°**  60° reference angle  Quadrant II

**Quadrant I**  60°  I am my own reference angle

**Quadrant III**  240°  60° reference angle

**Quadrant IV**  315°  45° reference angle

**Examples: Draw the angle in standard position and then find the reference angle.**

*Find a Coterminal ∠*

**a) 150°** Q2

180° – 150°
= 30° Reference ∠

**b) $\dfrac{7\pi}{6}$** Q3    R – π

$\dfrac{7\pi}{6} - \dfrac{\pi \cdot 6}{1 \cdot 6}$

$\dfrac{7\pi}{6} - \dfrac{6\pi}{6} = \dfrac{\pi}{6}$

**c) –100°**

+360
260° Q3

260 – 180°
= 80°

**Find the reference angle for each angle with the given measure.**

**d) –35°**

+360
325° Q4

360 – 325
= 35°

**e) 245°** Q3

245° – 180°
= 65°

**f) –510°**

+360
–150°
+360
210° Q3

210 – 180°
= 30°

**g) $\dfrac{4\pi}{9}$** Q1

$\dfrac{4\pi}{9}$

**h) $\dfrac{-8\pi}{5} + \dfrac{10\pi}{5}$**

$+ \dfrac{2\pi \cdot 5}{1 \cdot 5}$

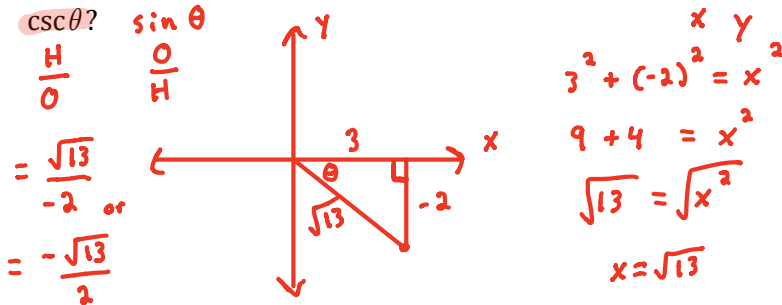$\dfrac{2\pi}{5}$ Q1

$\dfrac{2\pi}{5}$

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## Reference Triangle
- Formed by "dropping" a perpendicular from the terminal ray of a standard position angle to the x-axis.

$$y^+$$
$$^-x \quad \theta \quad x^+$$
$$y^-$$
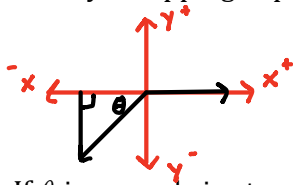
**Example 1:** If $\theta$ is an angle in standard position and P(-3, 4) is a point on the terminal side of $\theta$, what is the value of $\cos\theta$?

$$\frac{A}{H} = \frac{-3}{5}$$

$$\begin{array}{c} x \quad y \\ (-3)^2 + (4)^2 = x^2 \\ 9 + 16 = x^2 \\ \sqrt{25} = \sqrt{x^2} \\ x = 5 \end{array}$$
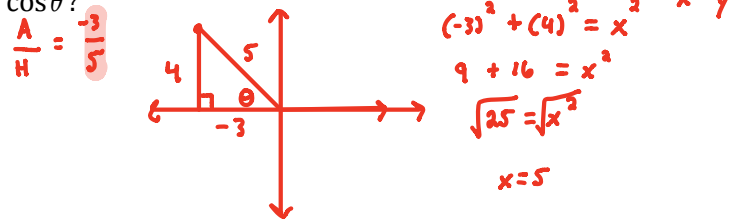
**Example 2:** If $\theta$ is an angle in standard position and P(3, -2) is a point on the terminal side of $\theta$, what is the value of $\csc\theta$?

$$\sin\theta$$
$$\frac{H}{O} \qquad \frac{O}{H}$$

$$= \frac{\sqrt{13}}{-2} \text{ or}$$

$$= -\frac{\sqrt{13}}{2}$$

$$\begin{array}{c} x \quad y \\ 3^2 + (-2)^2 = x^2 \\ 9 + 4 = x^2 \\ \sqrt{13} = \sqrt{x^2} \\ x = \sqrt{13} \end{array}$$

## On Your Own:

1. If $\theta$ is an angle in standard position and P(-4, 3) is a point on the terminal side of $\theta$, what is the value of $\sin\theta$?

2. If the terminal side of $\theta$ passes through point (-8, -6), what is the value $\cos\theta$?

**Sketch the angle in standard position in the coordinate plane that passes through each given point, and find all six trigonometric ratios for that point.**

3) (7, 24)

4) (8, 15)

5) $(-3, 3\sqrt{3})$

$$\begin{array}{c} x \quad y \\ \sin\theta = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \\ \cos\theta = \frac{-3}{6} = \frac{-1}{2} \qquad \sec\theta = \frac{-2}{1} = -2 \\ \tan\theta = \frac{3\sqrt{3}}{-3} = -\sqrt{3} \qquad \cot\theta = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} \end{array}$$

$$(-3)^2 + (3\sqrt{3})^2 = x^2$$
$$\sqrt{36} = \sqrt{x^2}$$
$$x = 6$$

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