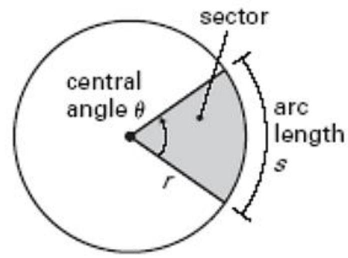
AFM | Name Key

**Unit 3 Day 3 Notes – Arc Length and Area of a Sector** | Date ______________________

The arc length, s, of a sector of a circle with radius r and central angle $\theta$ measured in radians, is

$$\boxed{\text{Arc length: } s = r\theta}$$




Examples:

1. Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30º.

$30^\circ \times \frac{\pi}{180}$

$\frac{30}{180} = \frac{1}{6} = \boxed{\frac{\pi}{6} = \theta}$

$s = r\theta$

$s = (10)\left(\frac{\pi}{6}\right)$

$s = 5.24$ m or $\frac{5\pi}{3}$ m

2. A central angle $\theta$ in a circle of radius 4 m is subtended by an arc of length 6 m. Find the measure of $\theta$ in radians and in degrees.

r s

$s = r\theta$

$\frac{6}{4} = \frac{(4)\theta}{4}$

$\theta = 1.5$ rad.

$\boxed{1 \text{ radian} = 57.3^\circ}$

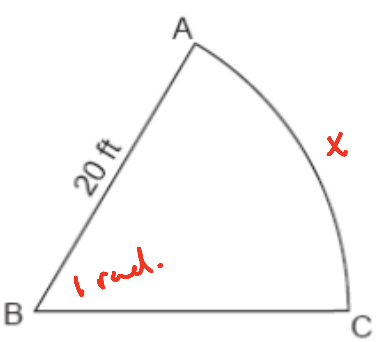
$1.5 \times 57.3$

$\theta = 85.95^\circ$ (degrees)

~~3.~~ Memphis, TN and New Orleans, LA lie approximately on the same meridian. Memphis has latitude 35ºN and New Orleans 30ºN. Find the distance between the two cities. (Radius of earth is 3960 miles)

**On Your Own:**

1. A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle *ABC* measuring 1 radian and radius $AB = 20$ feet. What is the length of arc *AC*, in feet?

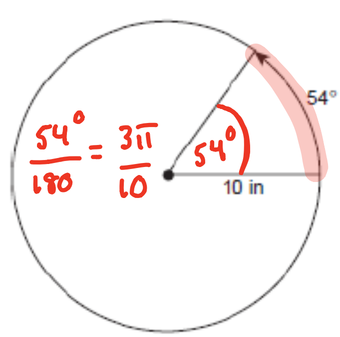



$s = r\theta$

$s = (20)(1)$

$s = 20$ ft.

2. A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the *nearest hundredth of an inch.*



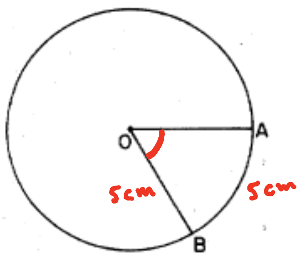

$s = (10)\left(\frac{3\pi}{10}\right)$

$s = 9.42$ in

or

$3\pi$

3. In circle O, the length of radius $\overline{OB}$ is 5 centimeters and the length of $\overset{\frown}{AB}$ is 5 centimeters. What is the measure of $\angle AOB$?




$s = r\theta$

$\frac{5}{5} = \frac{\cancel{5}\theta}{\cancel{5}}$

$\theta = 1$ rad.

4. An arc of a circle measures 30 centimeters ($s$) and the radius measures 10 centimeters ($r$). In radians, what is the measure of the central angle that subtends the arc?

$s = r\theta$

$\frac{30}{10} = \frac{\cancel{(10)}\theta}{\cancel{10}}$

$\theta = 3$ rad.

## Area of a Circular Sector

Do you recall the area of a circle? A sector of this circle with central angle $\theta$ (in radians) has an area that is a fraction of the area of the entire circle. Again, if angle is given in degrees, you must convert to radians.

$$A = \frac{1}{2}r^2\theta$$

Examples:

1. Find the area of a sector of a circle with a central angle of 45º if the radius is 2 m.

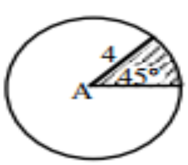
$A = (\frac{1}{2})(2)^2(\frac{\pi}{4})$ ↳ $\frac{\pi}{4}$ 1.57 $m^2$ or $\frac{\pi}{2}$ $m^2$

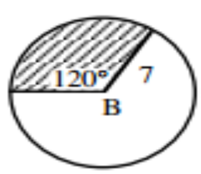
2. Find the radius of the circle if the area of a sector of a circle with a central angle of 4 radians is 2 m

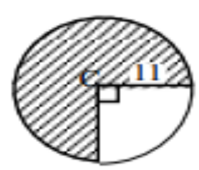
$A = (\frac{1}{2})(2)^2(4)$ 8 $m^2$

**On Your Own:**

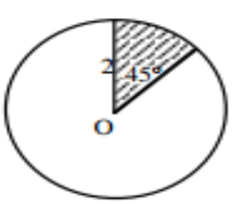
1. **Find the area of the shaded sector in each circle below. Points A, B and C are the centers.**

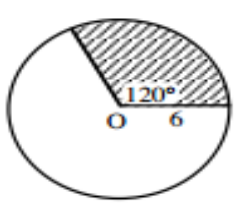
a) 

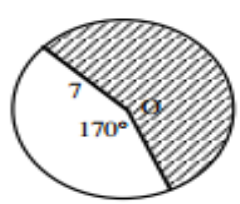

b) 

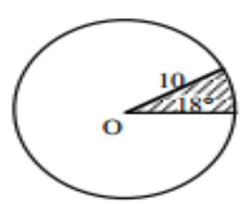

c) 


**Calculate the area of the following shaded sectors. Point O is the center of each circle.**

**2.** 


**3.** 


**4.** 


**5.** 

**Unit 3 Day 3 Notes – Cont.**

**You should be familiar with these types of problems:** A runner of a 4.2 mile race finished in 28 minutes and 4 seconds. What was the runner's average velocity in miles per hour?

$$speed = \frac{distance}{time}$$

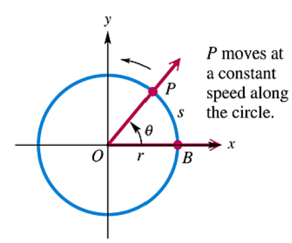
**<u>Linear and Angular Speed (Velocity)</u>**

Sometimes it is important to know how fast a point is moving (**Linear Speed**) or how fast a central angle is changing (**Angular Speed**). Radian measure and arc length can be applied to the study of *circular motion.*

Linear speed is measured in distance units per unit time (e.g. feet per second). The word linear is used because straightening out the arc traveled by the object along the circle results in a line of the same length, so that the usual definition of speed as distance over time can be used. Angular speed gives the rate at which the central angle swept out by the object changes as the object moves around the circle, and it is thus measured in radians per unit time.

> Think about 2 objects on a spinning disk, one being close to the center of the disk and one being close to the outside of the disk. Angular speed deals strictly with the angle. How long does each object take to move an angle of pi when the disk is spinning? It takes them the same amount of time, so they have the same angular speed.
>
> However, think about the actual speed of each object. The one that is further away from the center has to go a further distance to go around the circle than the one close to the center in the same amount of time, so it is going faster (linear speed). For this reason the radius (how far it is from the center) must be considered in the linear speed.

**Linear Speed (Velocity):** If $P$ is a point on a circle of radius $r$, and $P$ moves a distance $s$ on the circumference of the circle in an amount of time $t$, then the ***linear velocity***, $v$, of $P$ is given by the formula




$$v = \frac{s}{t} \qquad or \qquad \boxed{v = \frac{r\theta}{t}}$$

a) Example: A tire with radius of 9 inches is spinning at 80 revolutions per minute. Find the linear speed in feet per ~~minute~~ second.

rpm

$t = 60$ sec.

$\theta = 80 \cdot 2\pi$

$\theta = 160\pi$

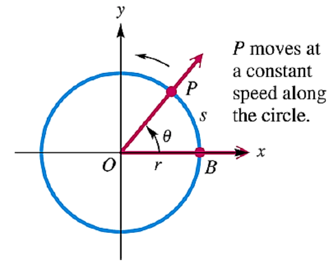
$$v = \frac{(9)(160\pi)}{60}$$

$v = 75.4$ ft/sec

or

$24\pi$ ft./sec.

**Angular Speed (Velocity)**: The measure of how fast an angle is changing, *angular velocity*, $\omega$(omega)




$$\boxed{\omega = \frac{\theta}{t}} \qquad where\ \theta\ is\ measured\ of\ angle\ in\ radians\ at\ time, t$$

b) Example: A tire with radius of 9 inches is spinning at 80 revolutions per minute. Find the angular speed of the tire in radians per second

$80 \cdot 2\pi = 160\pi$

$t = 60$

$\omega = \frac{160\pi}{60} = 8.38$ rad/sec

or $\frac{8\pi}{3}$ rad/sec.

c) Example: A mechanical arm rotates 1/3 of a rotation (revolution) in 0.25 seconds. Determine the angular speed in radians per second. (Note: One rotation is $2\pi$ radians—change to radians)