Math 2 Name: Key

Unit 3 Day 3 Notes – Solving Radical Equations Date:____________

Example 1: $(\sqrt{x})^2 = (3)^2$ | $(x^{1/2})^2 = (3)^2$

$x = 9$ | $x = 9$

To solve a radical equation, we essentially **square both sides** of the equation to 'undo' the radical.

$\sqrt{9} = 3$

$3 = 3$

Example 2: $(\sqrt{x})^2 = (-3)^2$

$x = 9$

$\sqrt{9} = -3$

$3 \neq -3$

No solution

Notice here that we solved correctly, but when we plug the solution back in, **the equation is not balanced/true.**

This is going to happen with radical equations; we will get an extraneous solution(s). In this context, it means that the solution is "mathematically correct, but not relevant or useful, as far as the original question is concerned".

Solving an Equation with Radicals

Step 1 **Change to radical form (if you can).**

Step 2 **Isolate the radical**. Make sure that one radical term is alone on one side of the equation.

Step 3 **Apply the power rule.** Raise both sides of the equation to a power that is the same as the index of the radical or the reciprocal of the rational exponent.

Step 4 **Solve** the resulting equation; if it still contains a radical, repeat Steps 2 and 3.

Step 5 **Check** all proposed solutions in the original equation.

Example 3:

$$-\sqrt{x+1} + 2 = -4$$

$-2 \quad -2$

$$\frac{-\sqrt{x+1}}{-1} = \frac{-6}{-1}$$

$$(\sqrt{x+1})^2 = (6)^2$$

$$x + 1 = 36$$

$-1 \quad -1$

$$\boxed{x = 35}$$

Example 4:

$$2(5x-1)^{\frac{1}{2}} - 2 = 0$$

$+2 \quad +2$

$$\frac{2(5x-1)^{1/2}}{2} = \frac{2}{2}$$

$$((5x-1)^{1/2})^2 = (1)^2$$

$$5x - 1 = 1$$

$+1 \quad +1$

$$\frac{5x}{5} = \frac{2}{5}$$

$$\boxed{x = \frac{2}{5}}$$

Example 5:

$$(3x+2)^{\frac{1}{3}} + 1 = 0$$

$-1 \quad -1$

$$((3x+2)^{1/3})^3 = (-1)^3$$

$$3x + 2 = -1$$

$-2 \quad -2$

$$\frac{3x}{3} = \frac{-3}{3}$$

$$\boxed{x = -1}$$

Example 6:

$$\frac{4(3x+5)^{\frac{2}{3}}}{4} = \frac{100}{4}$$

$$((3x+5)^{2/3})^{3/2} = (25)^{3/2}$$

raise both sides to the reciprocal of the exponent (flipping the fraction)

$$3x + 5 = 125$$

$-5 \quad -5$

$$\frac{3x}{3} = \frac{120}{3}$$

$$\boxed{x = 40} \checkmark$$

Example 7:

$$3(x+2)^{\frac{3}{4}} + 6 = 30$$

$-6 \quad -6$

$$\frac{3(x+2)^{3/4}}{3} = \frac{24}{3}$$

$$((x+2)^{3/4})^{4/3} = (8)^{4/3}$$

$$x + 2 = 16$$

$-2 \quad -2$

$$\boxed{x = 14}$$

Example 8:

$$\left((3x-1)^{\frac{1}{2}}\right)^{\cancel{2}} = \left((2x+5)^{\frac{1}{2}}\right)^{\cancel{2}}$$

$3x - \cancel{1} = 2x + 5$
$\cancel{+1} \quad\quad + 1$

$3x = \cancel{2x} + 6$
$-2x \quad \cancel{-2x}$

$\boxed{x = 6}$

Example 9:

$$\left(\sqrt[3]{2x+7}\right)^{\cancel{3}} = \left(\sqrt[3]{3x-2}\right)^{\cancel{3}}$$

$2x + 7 = 3x - \cancel{2}$
$+2 \quad\quad \cancel{+2}$

$2x + 9 = 3x$
$-2x \quad\quad -2x$

$\boxed{x = 9}$

Example 10:

$$(x)^2 = \left(\sqrt{x^2-5x+15}\right)^{\cancel{2}}$$

$\cancel{x^2} = \cancel{x^2} - 5x + 15$
$\cancel{-x^2} \quad \cancel{-x^2}$

$0 = -5x + \cancel{15}$
$-15 \quad\quad \cancel{-15}$

$\frac{-15}{-5} = \frac{-5x}{-5}$

$\boxed{x = 3}$ ✓

$3 = \sqrt{(3)^2 - 5(3) + 15}$

$3 = \sqrt{9}$

$3 \overset{✓}{=} 3$

Example 11:

$$\left(\sqrt{5-x}\right)^{\cancel{2}} = (x+1)^2 \rightarrow (x+1)(x+1)$$

$x^2 + x + x + 1$

	x	1
x	x^2	x
1	x	1

$\cancel{5} - \cancel{x} = x^2 + 2x + 1$
$\cancel{-5} \quad \cancel{+x} \quad\quad + x - 5$

$0 = x^2 + 3x - 4$

-4; 4, -1; 3 → change signs

$x \cancel{=} -4$ $\quad \boxed{x \overset{✓}{=} 1}$

$\sqrt{5+(+4)} = -4 + 1$

$\sqrt{9} = -3$

$3 \neq -3$

$\sqrt{5-1} = 1 + 1$

$\sqrt{4} = 2$

$2 \overset{✓}{=} 2$

Example 12:

$$\left(\sqrt{1-2x-x^2}\right)^{\cancel{2}} = (x+1)^2$$

$\cancel{1} - \cancel{2x} - \cancel{x^2} = x^2 + 2x + \cancel{1}$
$\cancel{-1} \quad \cancel{+2x} \quad \cancel{+x^2} \quad +x^2 + 2x \quad \cancel{-1}$

$0 = 2x^2 + 4x$

$0 = 2x(x+2)$

$\frac{\cancel{2}x = 0}{\cancel{2} \quad\; 2}$ $\quad\quad$ $x + 2 = 0$
$\quad\quad\quad\quad\quad\quad -2 \;\; -2$

$x \overset{✓}{=} 0$ $\quad\quad$ $x \cancel{=} -2$

$\sqrt{1-2(0)-(0)^2} = 0 + 1$

$\sqrt{1} = 1$

$1 \overset{✓}{=} 1$

$\sqrt{1-2(-2)-(-2)^2} = -2 + 1$

$\sqrt{1} = -1$

$1 \neq -1$

Example 13:

$$\sqrt{2x+3} + \cancel{x} + \cancel{1} = \cancel{1}$$

$\cancel{-x} \quad \cancel{-1} \quad \cancel{-1} - x$

$\left(\sqrt{2x+3}\right)^2 = (-x)^2$

$\cancel{2x} + \cancel{3} = x^2$
$\cancel{-2x} \quad \cancel{-3} \quad\quad -2x - 3$

$0 = x^2 - 2x - 3$

-3; -3, 1; -2 → change signs

$x \cancel{=} 3$ $\quad \boxed{x = -1}$

$\sqrt{2(3)+3} + 3 + 1 = 1$

$\sqrt{9} + 4 = 1$
$\quad\quad -4 \quad -4$

$\sqrt{9} = -3$

$3 \neq -3$

$\sqrt{2(-1)+3} \cancel{-1+1} = 1$

$\sqrt{1} = 1$

$1 \overset{✓}{=} 1$