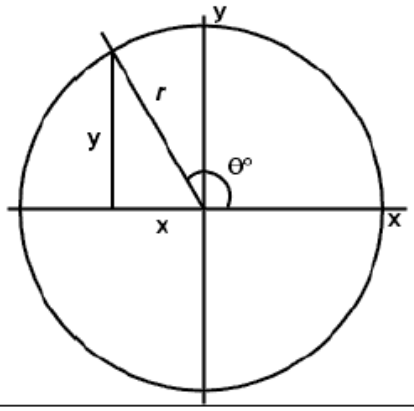
AFM
Unit 3 Day 4 Notes – The Unit Circle

Name Key
Date _____

The **Unit Circle** is the circle of radius 1 centered at the origin in the xy-plane.

The equation of the unit circle is $x^2 + y^2 = 1$

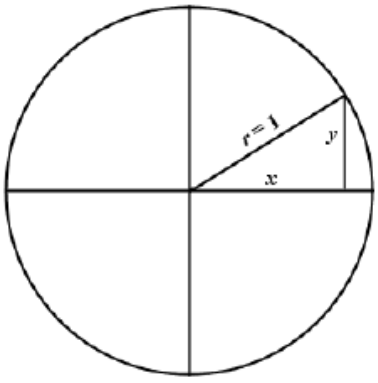
If the point (x, y) is on a circle with radius r, then:



$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

In a unit circle, the radius is one, so:



$$\cos\theta = \frac{x}{1} = x$$

$$\sin\theta = \frac{y}{1} = y$$

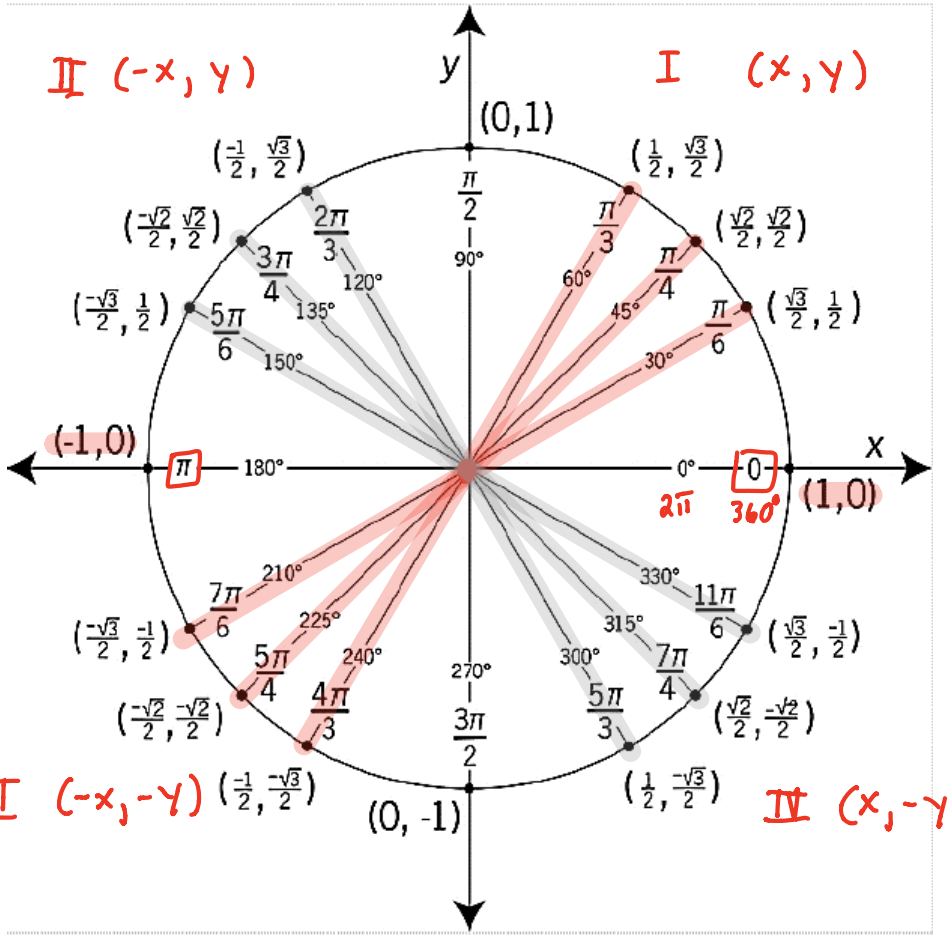
Using the unit circle makes finding sine and cosine (and other trig functions) simpler since $\cos\theta = x$ and $\sin\theta = y$

There are important values you need to know exact values of sine and cosine and be able to use those to find the other trig values.

θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

It may be helpful for you to look at the unit circle. You should memorize the first quadrant and then be able to figure out the other quadrants.

Unit Circle:



$\sin\theta = y$ | $\csc\theta = \frac{1}{y}$

$\cos\theta = x$ | $\sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ | $\cot\theta = \frac{x}{y}$

Things to remember about the unit circle:

- **Cos** = "x" values, **Sin** = "y" values, and **Tan** = $\frac{y}{x}$ values.
- <u>All Students Take Classes</u> 1st Quadrant (_____), 2nd quadrant (_____),

  3rd Quadrant (_____), 4th quadrant (_____).

II	I
(−, +)	(+, +)
III	**IV**
(−, −)	(+, −)

(y-axis: II / I; x-axis: III / IV)

- The main points you need to learn are in the first quadrant because everything is derived from the 1st quadrant.

Examples:

Find the exact trig value for the following:

1. a) $\sin 0$ (1, 0) x y $= 0$ b) $\cos 0$ $= 1$

2. a) $\sin(-\pi)$ Find a coterminal ∠ $+2\pi$ $\sin(\pi)$ $= 0$ (-1, 0) x y b) $\cos(-\pi)$ $+2\pi$ $\cos(\pi)$ $= -1$

Find the exact value of the trigonometric function at the given real number. Do NOT use your calculator!

3. a) $\sin(\pi/2)$ (x, y) $(0, 1)$ $= 1$ b) $\sin(3\pi/2)$ $(0, -1)$ $= -1$

4. a) $\cos(7\pi/3)$ $-\frac{6\pi}{3}$ $3 \cdot -2\pi / 1 \cdot 3$ $\cos \pi/3$ $= \frac{1}{2}$ $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ flipped b) $\sec(7\pi/3)$ $\sec \pi/3$ $= \frac{2}{1} = 2$

5. a) $\sec(11\pi/3)$ $-\frac{2\pi \cdot 3}{1 \cdot 3}$ $-6\pi/3$ $(\frac{1}{x})$ $\sec(5\pi/3)$ $= \frac{1}{\frac{1}{2}} = 1 \cdot \frac{2}{1} = 2$ $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$ b) $\csc(11\pi/3)$ $-6\pi/3$ $(\frac{1}{y})$ $\csc(5\pi/3)$ $= \frac{1}{-\frac{\sqrt{3}}{2}} = 1 \cdot -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

6. a) $\tan(-\pi/4)$ $+\frac{2\pi \cdot 4}{1 \cdot 4}$ $+8\pi/4$ $(\frac{y}{x})$ $\tan(7\pi/4)$ $= \frac{-\sqrt{x}/x}{\sqrt{x}/x} = -1$ $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ b) $\cot(-\pi/4)$ $(\frac{x}{y})$ $\cot(7\pi/4)$ $= \frac{\sqrt{x}/x}{-\sqrt{x}/x} = -1$

7. From the information given, determine the quadrant in which the point lies if $\cos t > 0$ and $\tan t < 0$.

II	I
$(-x, y)$	(x, y)
$(-x, -y)$	$(x, -y)$
III	IV

x is positive: 1 + 4 $\frac{y}{x}$ is negative: 2 + 4 Q4

On Your Own:

1. From the information given, find the quadrant in which the terminal point determined by t lies.

(a) $\sin t > 0$ and $\cos t < 0$

y is positive: 1 + 2 x is negative: 2 + 3 Q2

(b) $\tan t > 0$ and $\sin t < 0$

$\frac{y}{x}$ is positive: 1 + 3 y is negative: 3 + 4 Q3

2. In what quadrant is…

a) $\sin\theta > 0$ and $\cos\theta < 0$

y is positive: 1 + 2 x is negative 2 + 3 Q2

b) $\sec\theta < 0$ and $\cot\theta < 0$

$\frac{1}{x}$ is negative: 1 + 4 $\frac{x}{y}$ is negative: 2 + 4 Q4

c) $\csc\theta < 0$ and $\cos\theta > 0$

$\frac{1}{y}$ is negative: 3 + 4 x is positive: 1 + 4 Q4

d) all trig functions are negative?

Never