Math 2 Name: Key

Unit 3 Day 5 Notes – Types of Variation Date:____________

- **DIRECT VARIATION:** Linear function with a y-intercept of 0. In a direct variation, both of the quantities are either increasing or both are decreasing.

- There are two methods for solving a direct variation problem:

  1) Equation of Variation: $y = kx$ where k is called the **constant of variation**

  2) Proportion: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

#1: The distance that a body near Earth's surface will fall from rest varies directly as the **square** of the number of seconds it has been falling. If a boulder falls from a cliff a distance of 122.5 m in 5 *seconds*, approximately how far will it fall in 8 *seconds*?

Method 1

$y = kx^2$

$122.5 = k(5)^2$

$\frac{122.5}{25} = \frac{25k}{25}$

$\boxed{k = 4.9}$ (1)

$\boxed{y = 4.9x^2}$ (2)

$y = 4.9(8)^2$

$\boxed{y = 313.6 \text{ m}}$ (3)

Method 2

$\frac{y_1}{(x_1)^2} = \frac{y_2}{(x_2)^2}$

$\frac{122.5}{(5)^2} = \frac{y_2}{(8)^2}$

$\frac{25y_2}{25} = \frac{7840}{25}$

$\boxed{y_2 = 313.6 \text{ m}}$

- **JOINT VARIATION:** more than two quantities in a **direct variation** relationship

- Equation of Variation: $y = kxz$ where k is called the **constant of variaton**

#2: If y *varies jointly as* x *and* z, and $y = \frac{1}{2}$ when $x = 27$ and $z = \frac{-2}{3}$, find y when $x = 9$ and $z = 18$

$y = kxz$

$\frac{1}{2} = k(27)(-\frac{2}{3})$

$\frac{1/2}{-18} = \frac{-18k}{-18}$

$k = -\frac{1}{36}$ (1)

(2) $y = -\frac{1}{36}xz$

$y = (-\frac{1}{36})(9)(18)$

$y = \frac{-9}{2}$ (3)

- **INVERSE VARIATION:** Rational function with vertical and horizontal asymptotes. In an inverse variation, one of the quantities is increasing while the second quantity is decreasing.

- Equation of Variation: $y = \frac{k}{x}$ where k is called the **constant of variation**

#3: The time of a trip varies inversely as the speed of the car. If a car being driven at 55 *mph* takes 2 *hours* to get from Wake Forest to Greensboro, how fast is the car traveling if the trip takes 2.5 *hours*?

$y = \frac{k}{x}$

$55(2) = \left(\frac{k}{\cancel{55}}\right)\cancel{55}$

$k = 110$ (1)

(2) $y = \frac{110}{x}$

$\frac{2.5}{1} = \frac{110}{x}$

$\frac{2.5x}{2.5} = \frac{110}{2.5}$

$x = 44$ mph (3)

➤ **COMPOUND VARIATION:** Both Inverse and Direct Variation in the same problem

➤ Equation of Variation: $y = \frac{kx}{z}$ where k is called the **constant of variation**

#4: The volume of gas varies directly with Kelvin temperature and inversely with pressure. If a certain gas has a volume of $\boxed{342}$ *cubic meters* at a temperature of $\boxed{300}$ *Kelvin degrees* under a pressure of $\boxed{200}$ *KPa (kilopascals)*, what will be the volume of the same gas at a temperature of *320 Kelvin degrees* under a pressure of *400 kPA*?

$y = \frac{kx}{z}$

$\frac{342}{1} = \frac{k(300)}{200}$

$\frac{300k}{300} = \frac{68400}{300}$

$k = 228$ (1)

(2) $y = \frac{228x}{z}$

$y = \frac{228(320)}{400}$

$y = 182.4$ cubic meters (3)

➤ **State whether each equation represents a direct, inverse, joint or compound variation. Then state the constant of variation.**

1. $y = \frac{9}{x}$ $9 = k$ Inverse	2. $z = \boxed{5}xy$ Joint	3. $y = \frac{\boxed{8}x}{z}$ Compound	4. $y = 2x$ Direct	5. $xy = 12$ $y = \frac{12}{x}$ Inverse
6. $z = \frac{1xy}{15}$ $z = \boxed{\frac{1}{15}}xy$ Joint	7. $y = \boxed{\frac{3}{4}}xz$ Joint	8. $y = \boxed{\frac{1}{3}}x$ Direct	9. $z = \frac{x}{12y}$ $z = \boxed{\frac{1}{12}}x/y$ Compound	10. $y = \frac{1x}{5}$ $y = \boxed{\frac{1}{5}}x$ Direct

➤ **Write a function for each variation relationship:**

11. $\boxed{W}$ varies directly as the square of d. $W = kd^2$
$y = kx$

12. V varies inversely as J. $V = \frac{k}{J}$
$y = \frac{k}{x}$

13. V varies inversely as $\boxed{p}$ and directly as $\boxed{T}$. $V = \frac{kT}{p}$
$y = \frac{kx}{z}$

14. F varies jointly as A and the square of v. $F = kAv^2$
$y = kxz$

15. L varies directly as the fourth power of d and inversely as the square root of h.
$L = \frac{kd^4}{\sqrt{h}}$

Unit 3 Day 5 HW

Write an equation for each statement and then solve:

1. If y *varies directly as* x and $y = 15$ when $x = 3$, find y when $x = 12$.	2. If y *varies directly as* x and $x = 36$ when $y = 4$, find x when $y = 24$.	3. If y *varies directly as* x^2 and $y = 12$ when $x = 4$, find y when $x = 6$.
4. If y *varies inversely as* x and $y = 2$ when $x = 8$, find x when $y = 14$.	5. *If* y *varies inversely as* x and $x = 7$ when $y = 21$, find y when $x = 42$.	6. If y *varies inversely as* x^3 and $y = 6$ when $x = \frac{-3}{4}$, find y when $x = 3$.
7. Suppose y *varies jointly with* x *and* z. If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 14$ and $z = 8$.	8. Suppose z *varies jointly with* x *and* y. If $x = 3$ and $y = 2$ when $z = 12$, find z when $x = 4$ and $y = 5$.	9. Suppose m *varies jointly* as n *and* p. If $n = 4$ and $p = 5$ when $m = 60$, find m when $n = 12$ and $p = 2$.
10. Suppose that y *varies directly as* x *and inversely as* z. If $y = 5$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.	11. Suppose y *varies directly as* $\sqrt{x}$ and *inversely as* z. If $y = 10$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 10$.	12. Suppose x *varies directly as* y^3 *and inversely as* $\sqrt{z}$. If $x = 7$ when $y = 2$ and $z = 4$, find x when $y = 3$ and $z = 9$.

Determine the type of variation and then write an equation for each statement. Then solve.

13. The number (B) of bolts a machine can make *varies directly* as the time (T) it operates. If the machine can make 6578 *bolts* in 2 *hours*, how many bolts can it make in 5 *hours*?

14. The number of cooks needed to prepare lunch *varies inversely* with the time. If it takes 9 *cooks four hours* to prepare a school lunch, how long would it take 8 *cooks* to prepare the lunch?

15. The current (I) in an electrical conductor *varies inversely* as the resistance (r) of the conductor. If the current is 2 *amperes* when the resistance in 960 *ohms*, what is the current when the resistance is 480 *ohms*?

16. Cheers *varied jointly* as the number of fans and the **square** of the jubilation factor. If there were 100 *cheers* when the number of fans was 100 and the jubilation factor was 4, how many cheers were there when there were only 10 *fans* whose jubilation factor was 20?

17. The volume of a cone *varied jointly* as the height of the cone and the area of the base. If a cone has a volume of 140 cm^3 when the height is 15 cm and the area of the base is 28 cm^2, find the volume of a cone with a height of 7 cm and a base area of 12 cm^2.

18. The number of girls *varies directly* as the number of boys and *inversely* as the number of teachers. When there were 50 *girls*, there were 10 *boys* and 20 *teachers*. How many boys were there when there were 10 *girls* and 100 *teachers*?

19. A pitcher's earned run average (ERA) *varies directly* as the number of earned runs allowed and *inversely* as the number of innings pitched. Joe Price had an ERA of 2.55 when he gave up 85 *earned runs in* 300 *innings*. What would be his ERA if he gave up 120 *earned runs in* 600 *innings*?

20. The maximum load that a cylindrical column with a circular cross section can hold *varies directly* as the fourth power of the diameter and *inversely* as the square of the height. A 9 *meter column* with a 2 *meter diameter* will support 64 *metric tons*. How many metric tons can be supported by a column 9 *meters high* and 3 *meters in diameter*?