

Long Division:

Examples:

1. Divide  $x^2 + 2x - 30$  by  $x - 5$

$$\begin{array}{r}
 x + 7 \\
 x - 5 \overline{) x^2 + 2x - 30} \\
 \underline{-x^2 + 5x} \phantom{-30} \\
 7x - 30 \\
 \underline{-7x + 35} \\
 5
 \end{array}$$

5  
remainder

$$\boxed{x + 7 + \frac{5}{x - 5}}$$

$$\begin{aligned}
 \frac{x^2}{x} &= x(x-5) \\
 &= x^2 - 5x \\
 &\text{change the signs} \\
 &= -x^2 + 5x \\
 \hline
 \frac{7x}{x} &= 7(x-5) \\
 &= 7x - 35 \\
 &\text{change signs} \\
 &= -7x + 35
 \end{aligned}$$

2. Divide  $2x^2 - 19x + 24$  by  $x - 8$

$$\begin{array}{r}
 2x - 3 \\
 x - 8 \overline{) 2x^2 - 19x + 24} \\
 \underline{-2x^2 + 16x} \phantom{+24} \\
 -3x + 24 \\
 \underline{3x - 24} \\
 0
 \end{array}$$

$$\boxed{2x - 3}$$

$$\begin{aligned}
 \frac{2x^2}{x} &= 2x(x-8) \\
 &= 2x^2 - 16x \\
 &\text{change signs} \\
 &= -2x^2 + 16x
 \end{aligned}$$

$$\begin{aligned}
 \frac{-3x}{x} &= -3(x-8) \\
 &= -3x + 24 \\
 &\text{change signs} \\
 &= 3x - 24
 \end{aligned}$$

3. Divide  $x^3 - 4x^2 + 3x + 2$  by  $x + 2$

$$\begin{array}{r}
 x^2 - 6x + 15 \\
 x+2 \overline{) x^3 - 4x^2 + 3x + 2} \\
 \underline{-x^3 - 2x^2} \phantom{+ 3x + 2} \\
 -6x^2 + 3x \phantom{+ 2} \\
 \underline{6x^2 + 12x} \phantom{+ 2} \\
 15x + 2 \\
 \underline{-15x - 30} \\
 -28
 \end{array}$$

remainder

$$\boxed{x^2 - 6x + 15 - \frac{28}{x+2}}$$

$$\begin{aligned}
 \frac{x^3}{x} &= x(x+2) \\
 &= x^3 + 2x^2 \\
 &= -x^3 - 2x^2 \\
 \hline
 \frac{-6x^2}{x} &= -6x(x+2) \\
 &= -6x^2 - 12x \\
 &= 6x^2 + 12x \\
 \hline
 \frac{15x}{x} &= 15(x+2) \\
 &= 15x + 30 \\
 &= -15x - 30
 \end{aligned}$$

4. Divide  $9x^3 - 18x^2 - x + 2$  by  $3x + 1$

$$\begin{array}{r}
 3x^2 - 7x + 2 \\
 3x+1 \overline{) 9x^3 - 18x^2 - x + 2} \\
 \underline{-9x^3 - 3x^2} \phantom{- x + 2} \\
 -21x^2 - x \phantom{+ 2} \\
 \underline{21x^2 + 7x} \phantom{+ 2} \\
 6x + 2 \\
 \underline{-6x - 2} \\
 0
 \end{array}$$

$$\boxed{3x^2 - 7x + 2}$$

$$\begin{aligned}
 \frac{9x^3}{3x} &= 3x^2(3x+1) \\
 &= 9x^3 + 3x^2 \\
 &= -9x^3 - 3x^2 \\
 \hline
 \frac{-21x^2}{3x} &= -7x(3x+1) \\
 &= -21x^2 - 7x \\
 &= 21x^2 + 7x \\
 \hline
 \frac{6x}{3x} &= 2(3x+1) \\
 &= 6x + 2 \\
 &= -6x - 2
 \end{aligned}$$

\* When missing terms, replace with zeros!

5. Divide  $x^3 + 27$  by  $x + 3$

$$(x^3 + 0x^2 + 0x + 27) \div (x+3)$$

$$\begin{array}{r} x^2 - 3x + 9 \\ x+3 \overline{) x^3 + 0x^2 + 0x + 27} \\ \underline{-x^3 - 3x^2} \phantom{+ 0x + 27} \\ -3x^2 + 0x \phantom{+ 27} \\ \underline{-3x^2 + 9x} \phantom{+ 27} \\ 9x + 27 \\ \underline{-9x - 27} \\ 0 \end{array}$$

$$\boxed{x^2 - 3x + 9}$$

$$\begin{aligned} \frac{x^3}{x} &= x^2(x+3) \\ &= x^3 + 3x^2 \\ &= -x^3 - 3x^2 \\ \hline \frac{-3x^2}{x} &= -3x(x+3) \\ &= -3x^2 - 9x \\ &= 3x^2 + 9x \\ \hline \frac{9x}{x} &= 9(x+3) \\ &= 9x + 27 \\ &= -9x - 27 \end{aligned}$$

Remainder & Factor Theorem:

How to check whether a polynomial is a factor WITHOUT division:

6. Is  $x + 1$  a factor of  $x^3 + 4x^2 + x - 6$ ?

change sign

$$x = -1$$

$$\boxed{P(-1) = -4}$$

① STO → Button on Calc.

$x+1$  is NOT a factor

② Table on the Calc. (y=)

\* Remainder has to be zero to be a factor

7. Is  $x - 3$  a factor of  $x^3 - 4x^2 + x + 6$ ?

$$x = 3$$

$$\boxed{P(3) = 0}$$

$x-3$  IS a factor

How to use division to factor completely:

8. One factor of  $2x^3 + 5x^2 - 22x + 15$  is  $x - 1$ .