Math 2 Name: Key

Unit 3 Day 6 Notes – Graphing Rational Functions Date:_____

- A rational function is a function that can be written as the ratio of two polynomials where the denominator does not equal zero.

- $f(x) = \frac{p(x)}{q(x)}$ where $q(x) \neq 0$

❖ **Steps to graph a rational function:** $y = \frac{n}{x-h} + k$

(vertical stretch or shrink; $a > 0$; $0 < a < 1$; $a > 1$; shift (+) up or (-) down; keep sign; change sign; shift left or right (-))

1) Determine the location of the asymptotes based on the transformations:

   A) Vertical asymptotes are placed based on the **horizontal translation**: $x = h$

   B) Horizontal asymptotes are placed based on the **vertical translation**: $y = k$

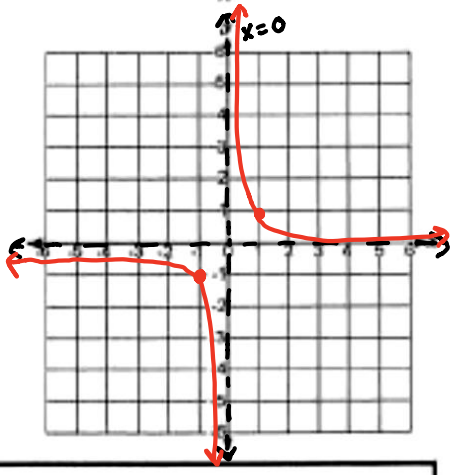
2) **Vertical Stretch or Compression**: n tells us how far the branches have been stretched from the asymptotes. We can use it to help us find out corner points to start our branches.

   Distance from asymptotes $= \sqrt{n}$

3) Look at the table on the calculator for other points and then sketch the two branches.

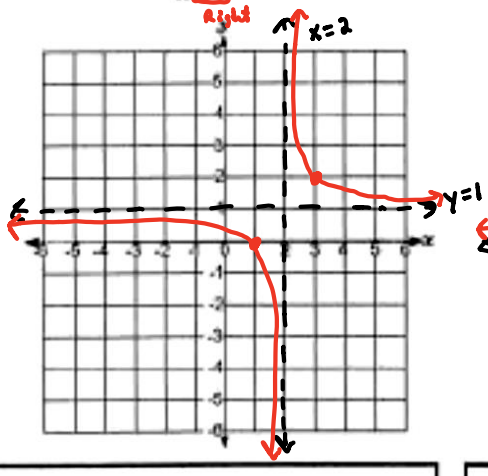
❖ Graph each of the following examples:

1. $y = \frac{1}{x}$ (parent)



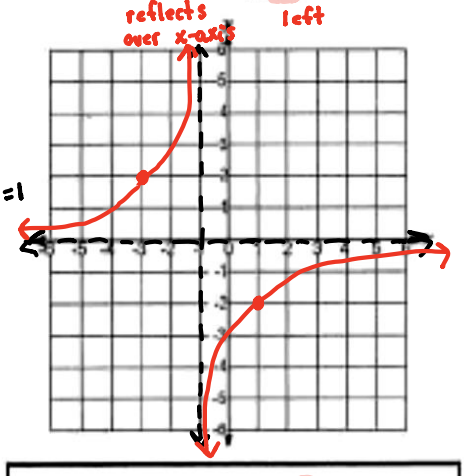
Equation of VA: $x = 0$
Equation of HA: $y = 0$
Describe translations: None
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

2. $y = \frac{1}{x-2} + 1$ (1st + 3rd Quadrants; up; right)



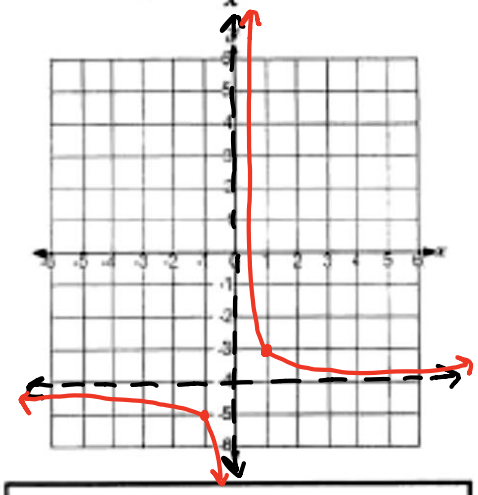
Equation of VA: $x = 2$
Equation of HA: $y = 1$
Describe translations: Right + Up
Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

3. $y = -\frac{4}{x+1} + 0$ (2nd + 4th Quadrants; vertical stretch; reflects over x-axis; left)



Equation of VA: $x = -1$
Equation of HA: $y = 0$
Describe translations: Reflects over the x-axis. Left. Vertical stretch
Domain: $(-\infty, -1) \cup (-1, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

4. $y = \frac{1}{x} - 4$



Equation of VA: $x = 0$

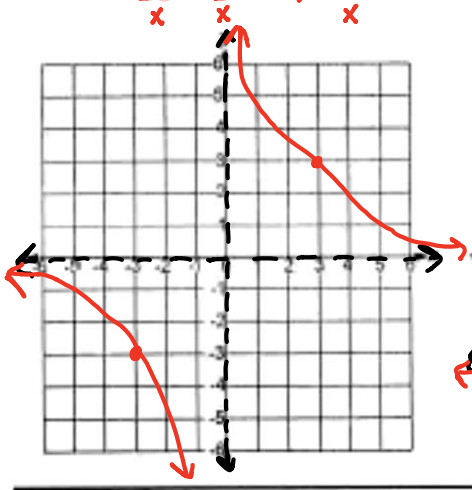
Equation of HA: $y = -4$

Describe translations: Down

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, -4) \cup (-4, \infty)$

5. $\frac{xy}{x} = \frac{9}{x}$ $y = \frac{9}{x}$



Equation of VA: $x = 0$

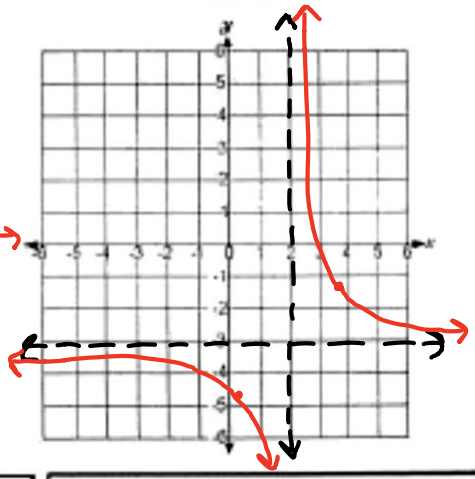
Equation of HA: $y = 0$

Describe translations: Vertical stretch

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

6. $y = \frac{3}{x-2} - 3$



Equation of VA: $x = 2$

Equation of HA: $y = -3$

Describe translations: Vertical stretch. Right. Down

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, -3) \cup (-3, \infty)$

7. Describe each graph as compared to the parent graph $y = \frac{1}{x}$.

$y = \frac{-2}{x-7} + 5$

The graph of this rational function has been translated right seven units and translated 5 units up. It has been vertically stretched by a factor of 2 and reflected across the x-axis. The graph is increasing from $-\infty$ to ∞. The function has a domain of $(-\infty, 7) \cup (7, \infty)$ and a range of $(-\infty, 5) \cup (5, \infty)$.

$y = \frac{7}{x+2} - 4$

The graph of this rational function has been translated left two units and translated 4 units down. It has been vertically stretched by a factor of 7. The graph is increasing from left to right. The function has a domain of $(-\infty, -2) \cup (-2, \infty)$ and a range of $(-\infty, -4) \cup (-4, \infty)$.

8. Write the equation of a rational function $y = \frac{1}{x}$ with following transformations:

A. Right 4 and Down 5 $(x-4)$ -5 $y = \frac{1}{x-4} - 5$

B. Left 3 and Up 2 and Reflected across $x-axis$. $y = \frac{-1}{x+3} + 2$

C. Left 6 and Vertically Stretched by a factor of 4. $(x+6)$ $y = \frac{4}{x+6}$

D. Right 5 and graph will be in II & IV quadrants $y = \frac{-1}{x-5}$

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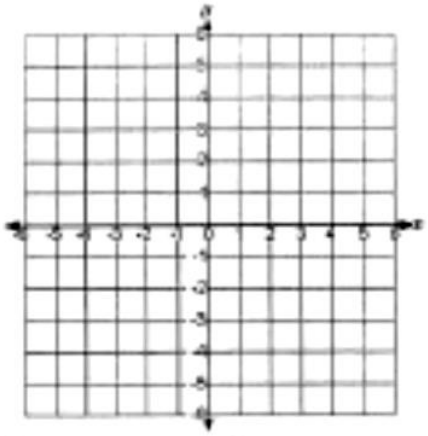
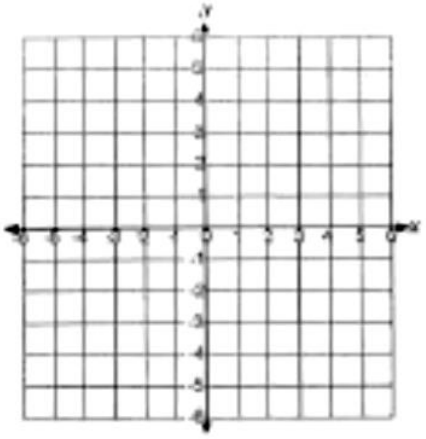
1. $y = \frac{1}{x} + 3$



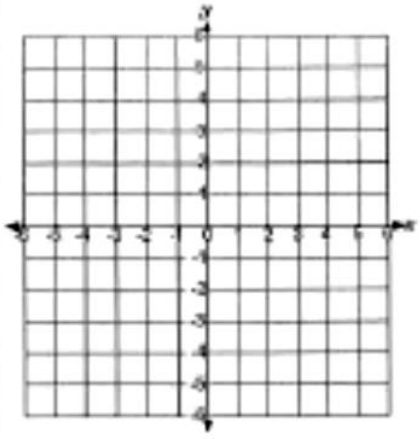
D: _____ R: _____

2. $y = \frac{1}{x-3}$



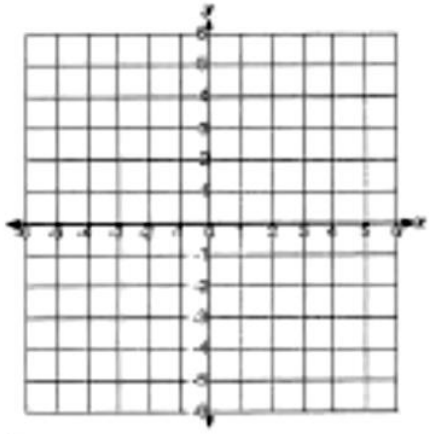
D: _____ R: _____

3. $y = \frac{1}{x+2} - 1$



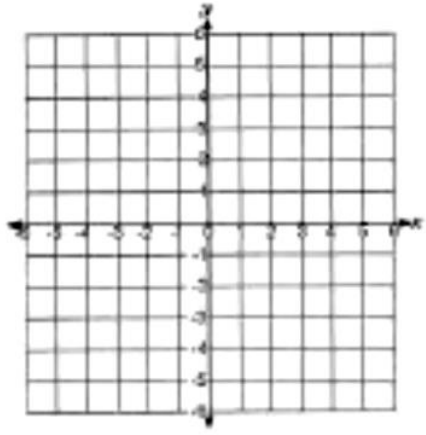
D: _____ R: _____

4. $y = \frac{2}{x}$



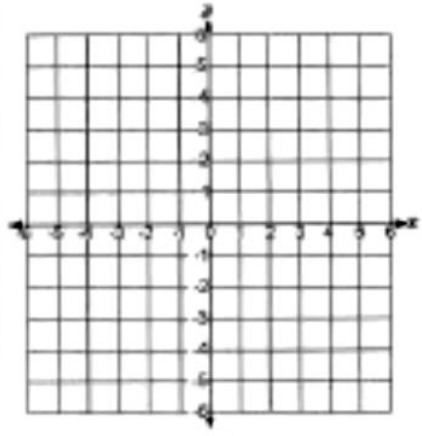
D: _____ R: _____

5. $y = \frac{3}{x+1}$



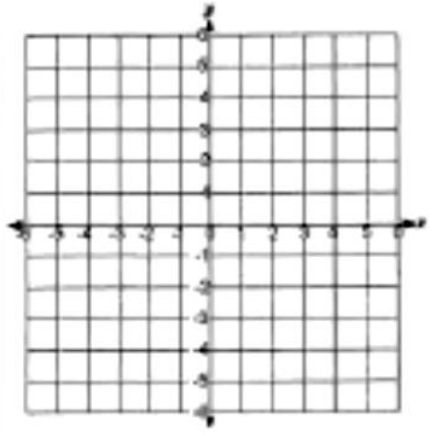
D: _____ R: _____

6. $y = \frac{4}{x-4} + 2$



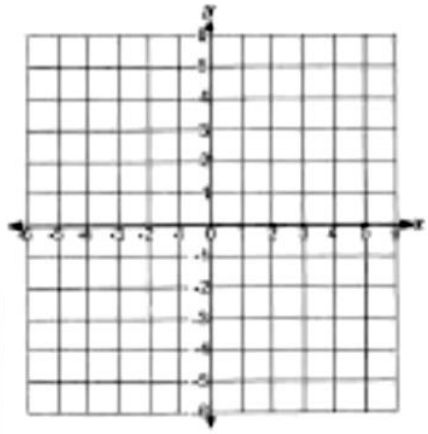
D: _____ R: _____

7. $y = -\frac{1}{x}$



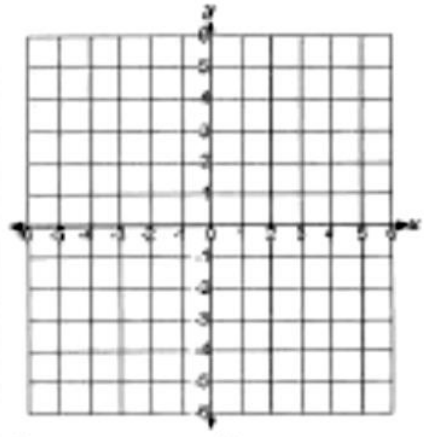
D: _____ R: _____

8. $y = -\frac{3}{x-2} + 1$



D: _____ R: _____

9. $y = -\frac{2}{x+1} - 2$



D: _____ R: _____

10. Consider the equation: $y = \frac{5}{x+1} - 2$

A) For what value is the function undefined (makes denominator = 0)? _____

B) What is the equation of the vertical asymptote? _____

C) What is the equation of the horizontal asymptote? _____

D) What is the domain of the function? _____

E) What is the range of the function? _____

F) What is the distance of the turning point from the intersection of the asymptotes? _____

G) In which quadrant is the center point located? _____

H) Graph the equation: