AFM Name Key

Unit 4 Day 3 Notes – Exponential Applications Date ____________

Compound Interest

Compound interest is interest compounded after different amounts of time periods, therefore becoming an exponential function.

Compound Interest is calculated by the formula

$$A(t) = P\left(1 + \left(\frac{r}{n}\right)\right)^{(nt)}$$

A(t) = accumulated amount after t years (principal + interest)

P= Principal investment (starting amount)

r = rate (percent changed to a decimal)

n = number of times per year compounded

t = time (in years)

Key words:

Yearly= 1 annually Monthly = 12 Quarterly = 4 Semi-annually= 2 Daily= 365

Ex 1: A sum of $1000 is invested at an interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, monthly and daily

P .12 = r t

n: 1, 2, 4, 12, 365

Annually:
$A = 1000\left(1 + \left(\frac{.12}{1}\right)\right)^{(1\cdot 3)}$
$A \approx \$1404.93$

Semiannually:
$A = 1000\left(1 + \left(\frac{.12}{2}\right)\right)^{(2\cdot 3)}$
$A \approx \$1418.52$

Quarterly:
$A = 1000\left(1 + \left(\frac{.12}{4}\right)\right)^{(4\cdot 3)}$
$A \approx \$1425.76$

Monthly:
$A = 1000\left(1 + \left(\frac{.12}{12}\right)\right)^{(12\cdot 3)}$
$A \approx \$1430.77$

Daily:
$A = 1000\left(1 + \left(\frac{.12}{365}\right)\right)^{(365\cdot 3)}$
$A \approx \$1433.24$

Ex 2: How much would you have to invest in order to have $700 after 2 years if the interest is compounded quarterly at an interest rate of 7.5%?

↳ P = ? A t

n = 4 ↳ .075 = r

$700 = P\left(1 + \left(\frac{.075}{4}\right)\right)^{(4\cdot 2)}$

(plug into calc.)

$\frac{700}{1.16} = \frac{1.16P}{1.16}$

$P \approx \$603.45$

"Euler's number," or ***e***, is a constant (similar to pi). "e" is approximately 2.71828. It is the limit of $(1 + 1/n)^n$ as n becomes large.

Used in cases where a quantity increases at a rate proportional to its value (bank account producing interest, population increasing as members reproduce, etc)

The **natural exponential function** is the exponential function $f(x) = e^x$ with base e.

We can use e for several types of word problems.

Continuously Compounded Interest

We can describe a function that compounds interest as frequently as possible as a function that compounds interest **continuously**. The small problem with continuous compounding is that if continuous means constantly, what value would we use for *n* in our compound interest formula? You will quickly figure out that there is no value for *n* that appropriately represents continuous. We can easily define *n* as 365 if we decide to compound interest once a day, but the value for *n* is intangible when you are referring to compounding interest continuously. So this all leads to the question – what formula can we use to compound interest on a principal amount continuously?

$$A = Pe^{rt}$$

A(t) = accumulated amount after t years (principal + interest)
P= Principal investment (starting amount)
r = rate (percent changed to a decimal)
t = time (in years)

Ex 3: Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

t P $.12 = r$

$$A = 1000e^{(.12 \cdot 3)}$$

$$A \approx \$1433.32$$

P = ?

Ex 4: Find the initial amount invested if after 3 years at an interest rate of 5.75% compounded continuously, you have $7500.

A t $r = .0575$

$$7500 = Pe^{(.0575 \cdot 3)}$$

(put in calc)

$$\frac{7500}{1.19} = \frac{1.19P}{1.19}$$

$$P \approx \$6302.55$$

Exponential Growth/Decay

A population that experiences exponential growth increases according to the model

$$n(t) = n_0 e^{rt}$$

n(t)= population at time t
n_0= initial population
r= *relative* growth rate (write as a decimal)
t= time - # of cycles **Must be in the same unit**

Ex 5: The initial bacterium count in a culture is 400. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per 2 hours.

a) Find a function that models the number of bacteria after t hours.

b) What is the estimated count after 8 hours?

Ex 6: A certain breed of rabbit was introduced into a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100, with a relative growth rate of 55% per year. What was the initial size of the rabbit population?

Half Life

If m_0 is the initial mass of a radioactive substance with half-life h, then the mass remaining at time t is modeled by the function : $m(t) = m_0 e^{-rt}$

Where relative decay rate $r = \dfrac{\ln 2}{n}$ and n = half-life cycle

***UNITS OF TIME ARE THE SAME THROUGHOUT THE PROBLEM!

Ex 7: Polonium-210 has a half life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

a) Find a function that models the amount of the sample remaining at time t

b) Find the mass remaining after one year

Ex 8: The half-life of a radioactive substance is one hundred fifty-three days. How many days will it take for seventy percent of the substance to decay?