AFM Name Key

Unit 4 Day 4 Notes – Logarithms Date ______________________

The **logarithm** with base a of a positive number x is defined as:

can't take logs of negative #'s

For $x > 0$ and $0 < a \neq 1$,
$y = \log_a x$ if and only if $x = a^y$

common log

In functional notation:
$f(x) = \log_a x$ is called the logarithmic function with base a

Because the logarithm with base e is used so frequently it has been given a special name **Natural logarithm** and abbreviation ln. The function is defined as:

$\log_e$

$f(x) = \log_e x = \ln x \quad x > 0$ is the natural logarithmic function.

By definition, the natural log function has an inverse function which is an exponential function.

$\ln_e x = y \leftrightarrow \boxed{e}^y = x$

Properties of Logarithms

1.	$\log_a 1 = 0$	because $a^0 = 1$
2.	$\log_a a = 1$	because $a^1 = a$
3.	$\log_{\cancel{a}} \cancel{a}^x = x$	Inverse Properties
4.	$\cancel{a}^{\log_{\cancel{a}} x} = x$	Inverse Properties

Properties of Natural Logarithms

All of the properties of logarithms listed above work for Natural Logarithms. There are also some special properties that apply only to the Natural Logarithms.

1.	$\ln 1 = 0$	because $e^0 = 1$
2.	$\ln e = 1$	because $e^1 = e$
3.	$\cancel{\ln} \cancel{e}^x = x$	Inverse Properties
4.	$\cancel{e}^{\cancel{\ln} x} = x$	Inverse Properties

Examples:
Evaluate the following:

a) $log_5 1 = 0$ $(5^0 = 1)$

b) $log_{\cancel{5}} \cancel{5}^8 = 8$

c) $log_{\cancel{5}} \cancel{5}^1 = 1$

d) $\cancel{5}^{log_{\cancel{5}} 12} = 12$

e) $\cancel{\ln} \cancel{e}^8 = 8$

f) $\ln\left(\frac{1}{e^2}\right) = \cancel{\ln} \cancel{e}^{-2} = -2$

g) $\ln 5 = 1.6094$

Solving for x.

Use the definition of logs to find x

a. $\log_3 x = 4$

$3^4 = x$

$\boxed{x = 81}$

b. $\log_x 125 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{125}$

$\boxed{x = 5}$

c. $\log_6 36 = x$

$6^x = 36$

$6^x = 6^2$

$\boxed{x = 2}$

d. $\log_4 x = \frac{1}{2}$

$\sqrt{4} \leftarrow 4^{(1/2)} = x$

$\boxed{x = 2}$

e. $\log_3\left(\frac{1}{9}\right) = x$

$3^x = \frac{1}{9^1}$

$3^x = 9^{-1}$

$3^x = 3^{-2}$

$\boxed{x = -2}$

f. $\log_3 243 = x$

$3^x = 243$

$3^x = 3^5$

$\boxed{x = 5}$

Evaluate

a. $\log_{10} 1 = x$

$10^x = 1$

$10^x = 10^0$

$\boxed{x = 0}$

b. $\cancel{\log_4} 4 = x$

$4^x = 4^1$

$\boxed{x = 1}$

c. $\cancel{\log_6 6}^5$

$x = 5$

d. $\log_4 2 = x$

$4^x = 2$

$2^{2x} = 2^1$

$\frac{2x}{2} = \frac{1}{2}$

$\boxed{x = \frac{1}{2}}$

e. $\cancel{10}^{\cancel{\log} 7}$

$x = 7$

f. $\log_9\left(\frac{1}{3}\right) = x$

$9^x = \frac{1}{3}$

$9^x = 3^{-1}$

$3^{2x} = 3^{-1}$

$\frac{2x}{2} = \frac{-1}{2}$

$\boxed{x = -\frac{1}{2}}$

g. $\cancel{\ln e}^3$

$x = 3$

Laws of Logarithms

Property	Definition	Example
Product	$\log_b mn = \log_b m + \log_b n$ (Condense ← / Expand →)	$\log_3 9x = \log_3 9 + \log_3 x$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$
Power	$\log_b m^p = p \cdot \log_b m$	$\log_2 8^x = x \cdot \log_2 8$
Equality	If $\log_b \boxed{m} = \log_b \boxed{n}$, then $m = n$.	$\log_8 \boxed{(3x-4)} = \log_8 \boxed{(5x+2)}$ so, $3x - 4 = 5x+2$

Examples: Use the laws of logs to rewrite each expression

a) $\log_2(6x)$ (1st, 2nd) $\underline{\log_2 6 + \log_2 x}$

b) $\log\sqrt{5} \rightarrow \log 5^{1/2} \rightarrow \frac{1}{2}\log 5$ or $\frac{\log 5}{2}$

c) $\log_5(x^3y^6)$ $\log_5 x^3 + \log_5 y^6 \rightarrow \underline{3\log_5 x + 6\log_5 y}$

d) $\ln(\frac{ab}{\sqrt[3]{c}}) \rightarrow \ln \frac{ab}{c^{1/3}} \rightarrow \ln a + \ln b - \ln c^{1/3}$

↳ $\ln a + \ln b - \frac{1}{3}\ln c$

or

$-\frac{\ln c}{3}$

Use the laws of logs to evaluate each expression

a) $\log_4(2) + \log_4(32)$

b) $\log_2(80) + \log_2(5)$

c) $-\frac{1}{3}\log_2 8$

a) $\log_4 (2 \cdot 32)$

$\log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

b) $\log_2 (80 \cdot 5)$

$\log_2 (400) = x$

Change of Base

$\frac{\log(400)}{\log(2)} \approx 8.6439$

c) $\log_2 8^{-\frac{1}{3}} = x$

$2^x = 8^{-1/3}$ (put in calc)

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$\boxed{x = -1}$