

I. Solving Logarithmic Equations

1. Condense (if needed) to a single logarithm
2. To eliminate the log, raise BOTH sides to the power that is on the base of the log
3. Use inverse properties to cancel the base and log. Then solve for x.

Examples:

a) Find the solution of the equation $-2 = \log(2) - \log(3+x)$

Division

$$-2 = \log_{10} \frac{2}{x+3}$$

$$10^{-2} = \frac{2}{x+3}$$

$$\frac{1}{100} = \frac{2}{x+3}$$

$$x+3 = \frac{200}{-3} - \frac{-3}{-3}$$

$$x = 197$$

b) Find the solution to the equation $\log_4(x-4) + \log_4 x = \log_4 5$

Mult.

$$\log_4 x(x-4) = \log_4 5$$

$$x^2 - 4x = 5$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$x-5=0 \Rightarrow x=5$ $x+1=0 \Rightarrow x=-1$

Change signs: $x=5$, $x=-1$

c) Find the solution to the equation $\ln 2x + \ln 4 = 3$

$$\ln 4(2x) = 3$$

$$4x = \frac{e^3}{4}$$

$$x = \frac{e^3}{16}$$

(exact answer)

$$x \approx 2.5107$$

(approximate)

Exponential

II. Solving Exponential Equations

1. Isolate the exponential expression on one side of the equation
2. Take the log of each side, then use the Laws of Logs to “bring down the exponent”
3. Solve for the variable

Examples:

d) Find the solution of the equation $3^{x+2} = 7$

$$\log 3^{x+2} = \log 7$$

$$(x+2) \log 3 = \log 7$$

$$\frac{x+2}{1} = \frac{\log 7}{\log 3}$$

$$x+2 = 1.7712$$

$$x = -0.2288$$

approximate answer

Exact answer: $\frac{\log 7}{\log 3} - 2$

e) Find the solution to the equation $8e^{2x} = 20$

$$e^{2x} = 2.5$$

$$\ln e^{2x} = \ln 2.5$$

$$\frac{2x}{2} = \frac{\ln 2.5}{2}$$

Exact answer

$$x \approx 0.4581$$

approximate answer