

How to Solve Rational Equations

1. Find a common denominators on each side of the equation
2. Simplify each side into one term, and then cross multiply.
3. Solve for the variable.
4. Check your answers

Example 1: Solve  $\left(\frac{y}{5} + \frac{y}{2} = 7\right)$  LCD: 10

$$2y + 5y = 70$$

$$\frac{7y}{7} = \frac{70}{7}$$

$$y = 10$$

Example 2: Solve  $\left(\frac{3x-2}{12} - \frac{1}{6} = \frac{1}{6}\right)$  LCD: 12

$$3x - 2 - 2 = 2$$

$$3x - 4 = 2$$

$$\frac{3x}{3} = \frac{6}{3} \quad x = 2$$

Example 3: Solve  $\left(\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}\right)$  LCD:  $3x^2$   
Restrictions:  $x \neq 0$

$$\frac{11x - x^2}{-11x + x^2} = \frac{-12}{+x^2 - 11x}$$

$$x^2 - 11x - 12 = 0$$

$$\frac{-12}{-11} \leftarrow \text{change signs}$$

$$x = 12, x = -1$$

Example 4: Solve  $\left(\frac{5}{2x} - \frac{2}{3} = \frac{1}{x} + \frac{5}{6}\right)$  LCD:  $6x$   
Restrictions:  $x \neq 0$

$$\frac{15 - 4x}{-5x} = \frac{6 + 5x}{-6x}$$

$$\frac{15 - 9x}{-15} = \frac{6}{-6}$$

$$\frac{9x}{-9} = \frac{-9}{-9} \quad x = 1$$

Example 5: Solve  $\left(x + \frac{6}{x} = -5\right)$  LCD:  $x$   
Restrictions:  $x \neq 0$

$$x^2 + 6 = -5x$$

$$x^2 + 5x + 6 = 0$$

$$\frac{6}{5} \leftarrow \text{change signs}$$

$$x = -3, x = -2$$

Example 6: Solve  $\left(\frac{5x-2}{x-4} = -3\right)$  LCD:  $x-4$   
Restrictions:  $x \neq 4$

$$5x - 2 = -3(x - 4)$$

$$5x - 2 = 3x + 12$$

$$2x = 14$$

$$\frac{2x}{2} = \frac{14}{2} \quad x = \frac{7}{1}$$

**Directions:** Simplify each rational expression. State all restrictions.

1.  $\frac{8x^2 - 10x + 3}{6x^2 + 3x - 3}$

2.  $\frac{5x}{2y+4} - \frac{6}{y^2+2y}$

3.  $\frac{x^2 - 16}{2x+8} \div \frac{(x-4)^2}{8x-32}$

4.  $3x - \frac{x^2 - 5x}{x^2 - 2}$

5.  $\frac{3x-6}{5x-20} \cdot \frac{x-8}{5x-10}$

6.  $\frac{y^2 - 25}{y^2 - 16} \div \frac{2y+10}{y^2 - 4y}$

7.  $\frac{8}{3x^3y} + \frac{4}{9xy^3}$

8.  $\frac{x^2}{x^2+2x+1} \div \frac{3x}{x^2-1}$

9.  $\frac{7}{5y+25} - \frac{4}{3y+15}$

**Directions:** Solve each equation. Check your solutions.

10.  $\left(\frac{x}{3} + \frac{x}{2} = 10\right)^6$  LCO: 6  
 $2x + 3x = 60$   
 $\frac{5x}{5} = \frac{60}{5}$   
 $x = 12$

11.  $\frac{-2}{x^2-2} = \frac{2}{x-4}$  Restrictions:  $x \neq 4$ ,  $x \neq \pm\sqrt{2}$   
 $2x^2 - 4 = -2x + 8$   
 $2x^2 + 2x - 12 = 0$   
 $\frac{2x^2 + 2x - 12}{2} = \frac{0}{2}$   
 $x^2 + x - 6 = 0$   
 $(x-2)(x+3) = 0$   
 $x = -3, x = 2$

12.  $\left(\frac{1}{2x} - \frac{2}{5x} = \frac{1}{2}\right)^{10x}$  LCO: 10x  
 $5 - 4 = 5x$   
 $\frac{1}{5} = \frac{5x}{5}$   
 $x = \frac{1}{5}$

13.  $\frac{-3}{x-1} = \frac{-2}{x+1}$  Restrictions:  $x \neq 1$ ,  $x \neq -1$   
 $-2(x-1) = -3(x+1)$   
 $-2x + 2 = -3x - 3$   
 $+3x - 2 = -3x - 3$   
 $+3x - 2 = -3x - 3$   
 $x = -5$

14.  $\left(\frac{1}{x} - \frac{1}{6} = \frac{4}{3x^2}\right)^{6x^2}$  LCO: 6x<sup>2</sup> Restrictions:  $x \neq 0$   
 $6x - x^2 = 8$   
 $x^2 - 6x + 8 = 0$   
 $(x-4)(x-2) = 0$   
 $x = 4, x = 2$

15.  $\frac{x+3}{x^2+3x-4} = \frac{x+2}{x^2-16}$  Restrictions:  $x \neq 4$ ,  $x \neq -4$ ,  $x \neq -1$   
 $(x+3)(x-4) = (x+2)(x-4)$   
 $x^2 - x - 12 = x^2 - 2x - 8$   
 $-x - 12 = -2x - 8$   
 $x = 4$  (no solution)