Math 3
Unit 5 Day 7 Notes – Graphing Rational Fucntions

Name: Key
Date:_____

Example 1: Simplify the following. State any restrictions on the variables.

a) $\dfrac{(x+1)(x-5)}{(x-5)(x^2-1)}$

$\dfrac{(x+1)(x-5)}{(x-5)(x+1)(x-1)} = \dfrac{1}{(x-1)}$ $x \neq 1$

Hole: $x=-1$ $(-1, -\frac{1}{2})$ $\frac{1}{-1-1} = \frac{1}{-2} = -\frac{1}{2}$

$x=5$ $(5, \frac{1}{4})$ $\frac{1}{5-1} = \frac{1}{4}$

b) $\dfrac{x^2+x-12}{x^2+7x+12}$

-12: $4, -3$; 12: $3, 4$; 7: $4, 3$

$\dfrac{(x+4)(x-3)}{(x+3)(x+4)} = \dfrac{(x-3)}{(x+3)}$ $x \neq -3$

Hole: $x=-4$ $(-4, 7)$ $\frac{-4-3}{-4+3} = \frac{-7}{-1} = 7$

Vertical Asymptotes: Where the bottom of a function equals zero.

Point of Discontinuity: A hole in the graph.

Example 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \dfrac{x^2-1}{x^2-6x+5}$.

(DOS) $= \dfrac{(x+1)(x-1)}{(x-1)(x-5)}$

-5: $-1, -5$; 5: $-1, -6$

$= \dfrac{(x+1)}{(x-5)}$ $\dfrac{1+1}{1-5} = \dfrac{2}{-4} = -\frac{1}{2}$

VA: $x=5$ (vertical dotted line)

Hole: $x=1$ $(1, -\frac{1}{2})$

$y = \#$ (horizontal line)

Example 3: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \dfrac{x^2-4}{x^2+5x+6}$

$= \dfrac{(x+2)(x-2)}{(x+3)(x+2)}$

6: $3, 2$; 5: $3, 2$

$= \dfrac{(x-2)}{(x+3)}$

Hole: $x=-2$ $(-2, -4)$

$\dfrac{-2-2}{-2+3} = \dfrac{-4}{1} = -4$

VA: $x=-3$

Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator.
Let m = degree of numerator and n = degree of denominator.

If...	Then the graph has...
$m < n$ $f(x) = \dfrac{x+4}{x^2+5x+4}$ $\dfrac{(x+4)}{(x+1)(x+4)} = \dfrac{1}{(x+1)}$; 4: $1, 4$; 5: $1, 4$	A horizontal asymptote at y = 0 V.A.: $x=-1$ Hole(s): $x=-4$ $(-4, -\frac{1}{3})$ H.A.: $y=0$ Domain: $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$
m = n $f(x) = \dfrac{x^2+5x+4}{4x^2-9}$ (DOS) $\dfrac{(x+1)(x+4)}{(2x-3)(2x+3)}$	A horizontal asymptote at the coefficient of m divided by the coefficient of n V.A.: $x=\frac{3}{2}$ $x=\frac{-3}{2}$ Hole(s): None H.A.: $y=\frac{1}{4}$ Domain: $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$
m > n $f(x) = \dfrac{x^2+5x+4}{x+4}$ $\dfrac{(x+1)(x+4)}{(x+4)} = (x+1)$	No horizontal asymptote V.A.: None Hole(s): $x=-4$ $(-4, -3)$ H.A.: None Domain: $(-\infty, -4) \cup (-4, \infty)$

Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

a) $f(x)=\dfrac{x^2+x-2}{x-1}$

$=\dfrac{(x-1)(x+2)}{(x-1)}$

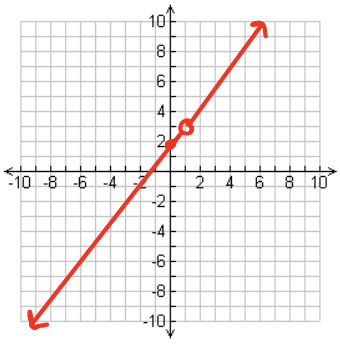
$=(x+2)$

HA: None

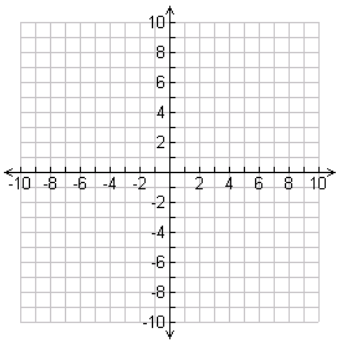
Hole: $x=1$ $(1,3)$ O

VA: None

Domain: $(-\infty,1)\cup(1,\infty)$



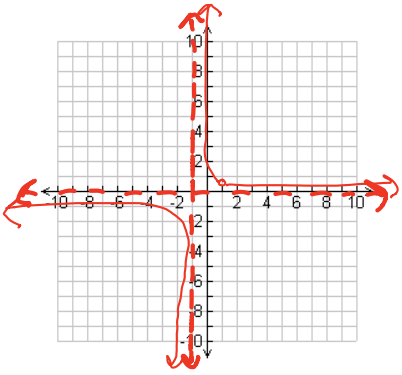
b) $f(x)=\dfrac{2x^2+3}{x+2}$



c) $f(x)=\dfrac{x-1}{x^2-1}$ (DOS)

$=\dfrac{x-1}{(x+1)(x-1)}$

$=\dfrac{1}{(x+1)}$



HA: $y=0$

Hole: $x=1$ $(1,\frac{1}{2})$

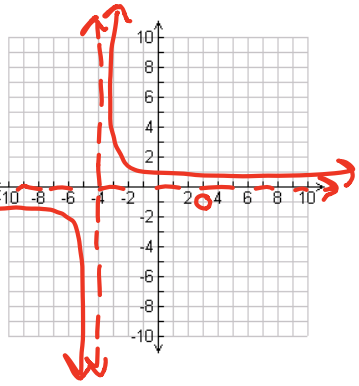
VA: $x=-1$

D: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

d) $f(x)=\dfrac{x-3}{x^2-7x+12}$

$=\dfrac{(x-3)}{(x-4)(x-3)}$

$=\dfrac{1}{(x-4)}$



HA: $y=0$

Hole: $x=3$ $(3,-1)$

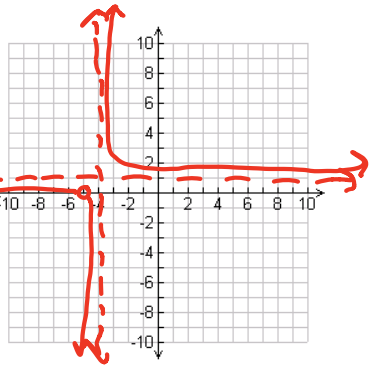
VA: $x=-4$

D: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

e) $f(x)=\dfrac{x^2+10x+25}{x^2+9x+20}$

$=\dfrac{(x+5)(x+5)}{(x+5)(x+4)}$

$=\dfrac{(x+5)}{(x+4)}$



HA: $y=1$

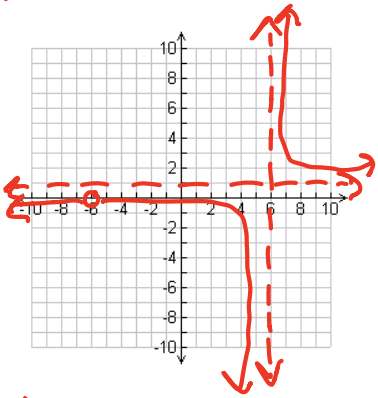
Hole: $x=-5$ $(-5,0)$

VA: $x=-4$

D: $(-\infty,-5)\cup(-5,-4)\cup(-4,\infty)$

f) $f(x)=\dfrac{x^2+12x+36}{x^2-36}$

$=\dfrac{(x+6)(x+6)}{(x+6)(x-6)}$



HA: $y=1$

Hole: $x=-6$ $(-6,0)$

VA: $x=6$

D: $(-\infty,-6)\cup(-6,6)\cup(6,\infty)$