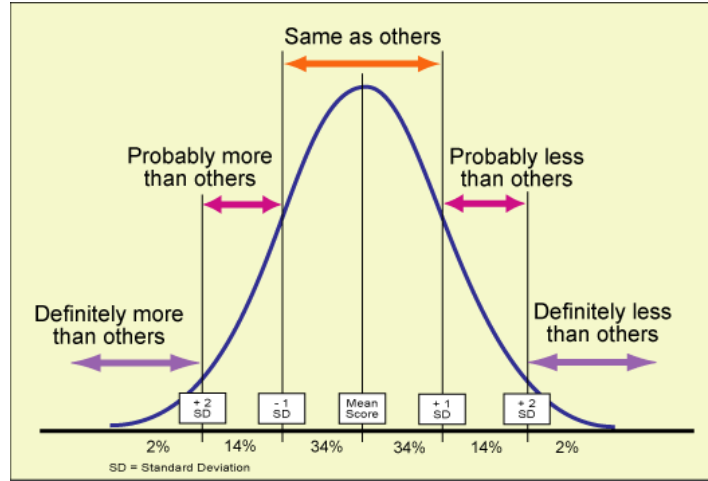
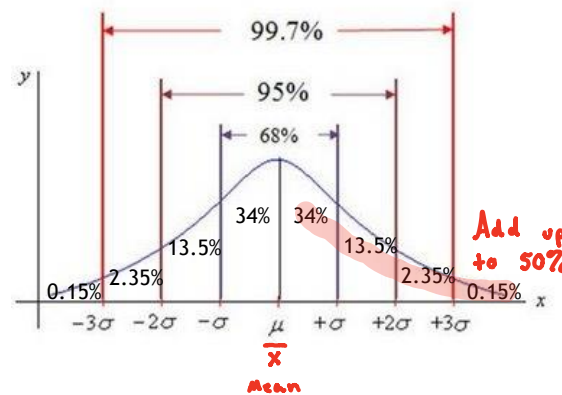


Normal Distribution



Distributions for large populations often have a bell-shaped distribution. Heights, clothing sizes, and test scores are a few examples. In fact, the bell-shaped curve is so common that it is called a Normal Distribution and a bell-shaped distribution is called a Normal Curve.

Most graphing calculators provide the normal distribution equation as a built-in function and you have to provide only the mean (μ - mu) and standard deviation (σ - lowercase sigma).
(or \bar{x})



The standard confidence intervals are as labeled in the above picture. The mean (μ) is in the very center of a normal curve.

68 % of the data fall within one standard deviation (σ) to the right and left of the mean.

95 % of the data fall within two standard deviations of the mean.

99.7 % of the data fall within three standard deviations of the mean.

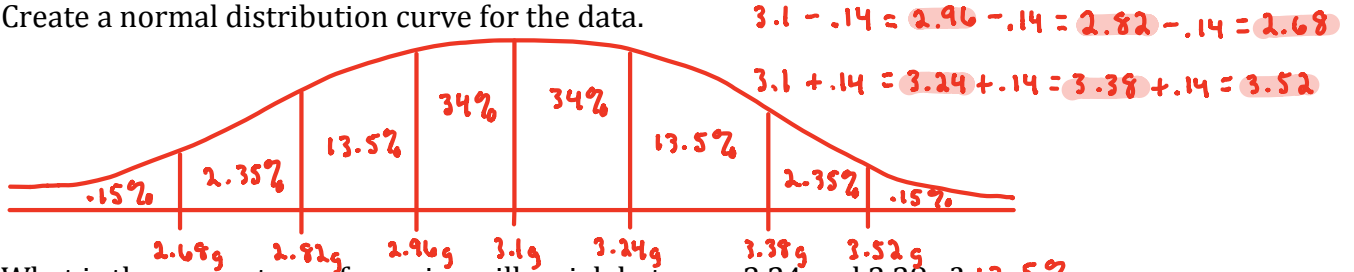
$$\begin{aligned}
 &34 \rightarrow \frac{1}{2} \text{ of } 68 \\
 &13.5 \rightarrow \frac{(95 - 68)}{2} \\
 &2.35 \rightarrow \frac{(99.7 - 95)}{2} \\
 &.15 \rightarrow \frac{(100 - 99.7)}{2}
 \end{aligned}$$

where other #'s come from

Example 1

A group of students weighs 500 U.S. pennies. They find that the pennies have normally distributed weights with a mean of 3.1g and a standard deviation of 0.14g.

a. Create a normal distribution curve for the data.



b. What is the percentage of pennies will weigh between 3.24 and 3.38 g? 13.5%

c. What is the percentage of pennies will weigh more than 3.24 g? 13.5 + 2.35 + .15 = 16%

d. How many pennies weigh less than 3.1g? 50% → .5(500) = 250 pennies

e. What is the probability that the weight of a penny will be within one standard deviation of the mean? 68% (below + above the mean)

Two standard deviations of the mean? 95%

Three standard deviations of the mean? 99.7%

Example 2

A class has test scores that are normally distributed with a mean of 82 and a standard deviation of 5. Give the percentage of all data values that fall within each interval.

a. Within two standard deviations of the mean 95%

b. Between the mean and two standard deviations above the mean 34 + 13.5 = 47.5%

c. Below the mean 50%

Z-Scores

The number of standard deviations from the mean are called z-scores. If the value is one standard deviation above the mean, it will have a z-value of 1. If a value is two standard deviations below the mean, it has a z-value of -2.

[z-score] can be a decimal

$$z = \frac{\text{data point } x - \mu}{\sigma} \rightarrow \bar{x} \rightarrow \text{mean}$$

Standard deviation

[Finding Data Point Using z-score]

$$\text{data point} = \mu + \sigma(z)$$

↓ standard deviation

↓ z-score

↓ mean

Example 3

The scores on a standardized test are normally distributed with a mean of 71 and a standard deviation of 3.8. Find the test scores for each of these z-values.

a. $z = 2$

$$t = 71 + 3.8(2)$$

$$t = 78.6$$

b. $z = -3$

$$t = 71 + 3.8(-3)$$

$$t = 59.6$$

c. $z = 1.5$

$$t = 71 + 3.8(1.5)$$

$$t = 76.7$$

d. $z = -2.8$

$$t = 71 + 3.8(-2.8)$$

$$t = 60.36$$

Find the z-score of the following test scores.

e. $80.5 = t$

$$z = \frac{80.5 - 71}{3.8}$$

$$z = 2.5$$

f. $90 = t$

$$z = \frac{90 - 71}{3.8}$$

$$z = 5$$

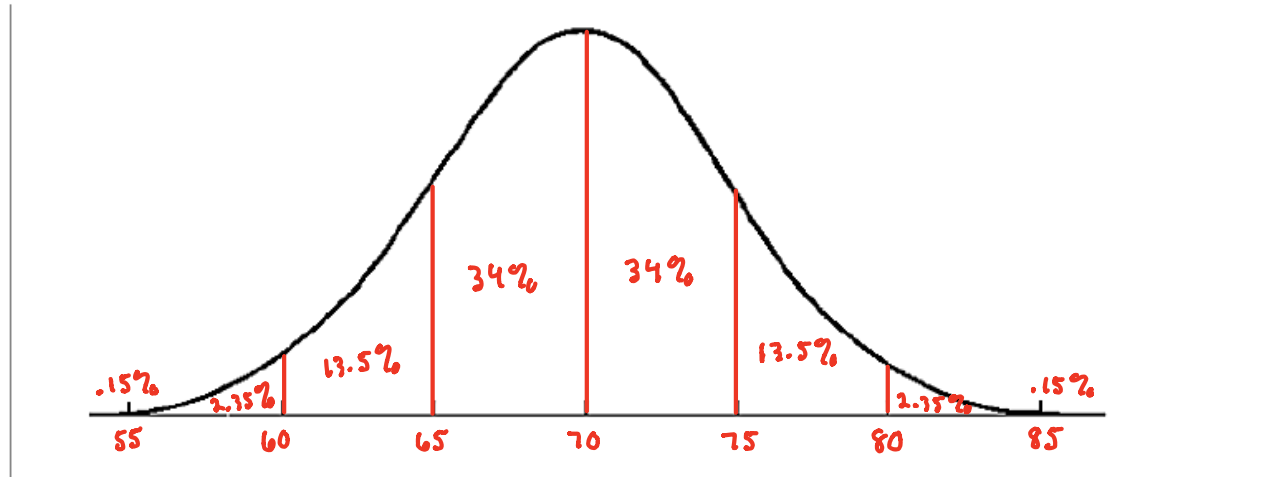
Example 4

~~What is the probability that a test score from the above situation will be between 67.2 and 78.6?~~

Empirical Rule

In a normal distribution, what percent of the values lie:

- below the mean? 50%
- above the mean? 50%
- within one standard deviation of the mean? 68%
- within two standard deviations of the mean? 95%
- within three standard deviations of the mean? 99.7%
- 2000 freshmen** at State University took a biology test. The scores were distributed normally with a **mean of 70** and a **standard deviation of 5**. Label the mean and three standard deviations from the mean.



Answer the following questions based on the data:

- What percentage of scores are between scores 65 and 75? 68%
- What percentage of scores are between scores 60 and 70?
 $34 + 13.5 = 47.5\%$
- What percentage of scores are between scores 60 and 85?
 $100 - 2.35 - .15 = 97.35\%$
- What percentage of scores is less than a score of 55? .15%
- What percentage of scores is greater than a score of 80?
 $.15 + 2.35 = 2.5\%$
- Approximately how many biology students scored between 60 and 70?
 $47.5\% \rightarrow .475(2000) = 950$ students
- Approximately how many biology students scored between 55 and 60?
 $2.35\% \rightarrow .0235(2000) = 47$ students

h) How many students fall within two standard deviations of the mean?

$$95\% \rightarrow .95(2000) = 1900 \text{ students}$$

i) Find the z-scores for the following test scores:

1. 63 $z = \frac{63 - 70}{5} = -1.4$

2. 81 $z = \frac{81 - 70}{5} = 2.2$

j) Find the test scores for the following values:

1. $z = -2.1$

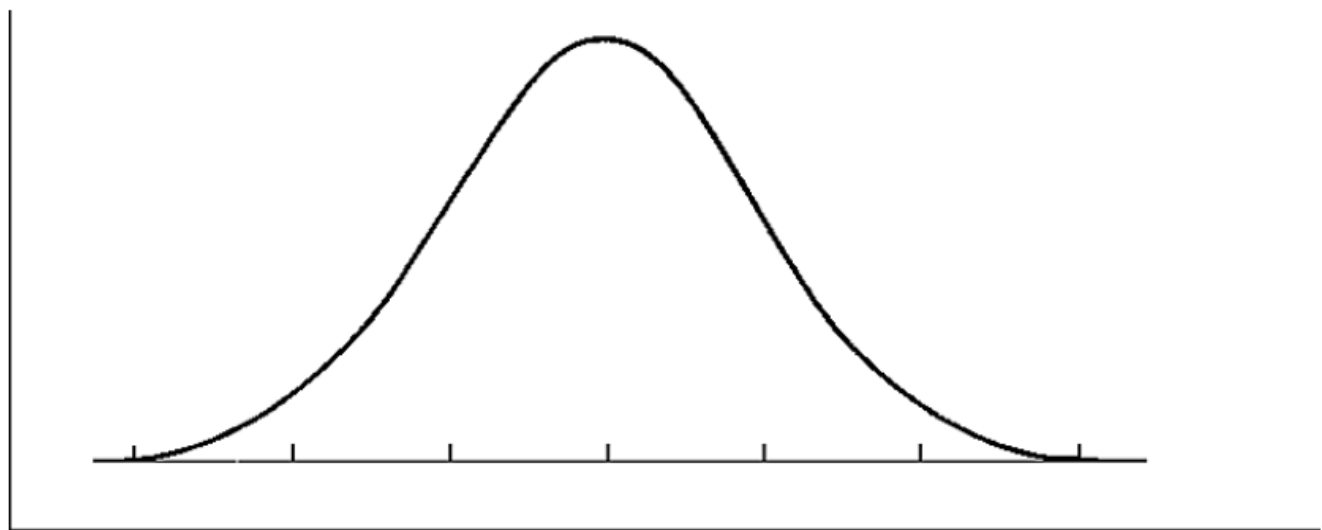
$$t = 70 + 5(-2.1) = 59.5$$

2. $z = 1.2$

$$t = 70 + 5(1.2) = 76$$

HW

7. 500 juniors at Central High School took the ACT last year. The scores were distributed normally with a mean of 24 and a standard deviation of 4. Label the mean and three standard deviations from the mean.



Answer the following questions based on the data:

- What percentage of scores are between scores 20 and 28?
- What percentage of scores are between scores 16 and 32?
- What percentage of scores are between scores 16 and 28?
- What percentage of scores is less than a score of 12?
- What percentage of scores is greater than a score of 24?
- Approximately how many juniors scored between 24 and 28?
- Approximately how many juniors scored between 20 and 28?
- Approximately how many juniors scored between 24 and 32?
- Approximately how many juniors scored between 16 and 20?
- Approximately how many juniors scored higher than 32?

Unit 6 Day 4 HW: Standard Deviation

1. The sum of the deviations about the mean always equals _____.
2. Given the following numbers, find the standard deviation:
43, 26, 92, 11, 8, 49, 52, 126, 86, 42, 63, 78, 91, 79, 86
3. a) Compute the standard deviation of the following test scores: 78, 78, 78, 78, 78.
b) What can be said about a data set in which all the values are identical?
4. The mean and standard deviation of a data set are mean = 10 and standard deviation = 4. Given the following numbers in the set, they are *within* how many standard deviations from the mean?
a) 14 b) 18 c) 20 d) 8 e) 10
5. You are filling out an application for college. The application requests either your ACT or SAT Math score. You scored 26 on the ACT composite and 650 on the SAT Math. On the ACT exam, the composite mean score is 21 with a standard deviation of 5, while the SAT Math has a mean score of 514 with a standard deviation of 113. Which test should you provide on the application? Explain your reasoning.
6. Suppose the frequency of each data item in the table below is doubled. What is the effect, if any, on the mean and standard deviation of the data?

| | | | |
|--------------|---|----|---|
| Item of Data | 3 | 6 | 7 |
| Frequency | 4 | 10 | 6 |

7. The cost of groceries when parents go shopping is normally distributed. Of the 26 different parents' visits this year, the mean amount of money they spent was \$196 with a standard deviation of \$12.
 - a) Make the curve to represent the normal distribution.
 - b) Of the parents surveyed, how many spent more than \$220?
 - c) What percentage of the parents spent less than \$184?