

| Conditional <br> Probability | is the probability of an event given that <br> another event heas occured |  |
| :---: | :--- | :--- |
| Formula for <br> $P(A \mid B)$ | $P(A \mid B)=\frac{P(A \cap B)}{P(B)}$ | $P(B \mid A)=P(A) \cdot P(B \mid A)$ |

## Examples:

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

$$
(13 / 52) \cdot(12 / 51)=1 / 17 \text { or } .059
$$

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

$$
(6 / 8) \cdot(2 / 7)=3 / 14 \text { or } .214
$$

3. In Mr. Jonas' homeroom, $70 \%$ of the students have brown hair, $25 \%$ have brown eyes, and $5 \%$ have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

$$
(.7) \cdot(.05)=7 / 200^{\text {or }} .035
$$

## Using Two-Way Frequency Tables to Compute Conditional Probabilities

Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table.

| $9^{\text {th }}$ or $10^{\text {th }}$ | Bus | Walk | Car | 0ther | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $11^{\text {th }}$ or $12^{\text {th }}$ | 106 | 30 | 70 | 4 | 260 |
| Total | 41 | 58 | 184 | 7 | 290 |

Suppose we randomly select one students
4. What is the probability that the student walked to school?

$$
88 / 500=22 / 125
$$

5. $\mathrm{P}\left(9^{\text {th }}\right.$ or $10^{\text {th }}$ grader $)$

$$
210 / 500=21 / 50
$$

6. P (rode the bus OR $11^{\text {th }}$ or $12^{\text {th }}$ grader)

$$
147 / 500+290 / 500-41 /_{500}=396 /_{500}=99 / 125
$$

7. What is the probability that a student is in 11th or 12th grade given that they rode in a car to school? $184 / 254=92 / 127$
8. What is $\mathrm{P}($ Walk $\mid 9$ th or 10 th grade $)$ ? $30 / 210{ }^{1} \mathrm{I}_{7}$

Example: The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

|  | Vanilla | Chocolate | Total |
| :---: | :---: | :---: | :---: |
| Adult | 52 | 41 | 93 |
| Child | 26 | 105 | 631 |
| Total | 78 | 146 | 224 |

9. Find $P$ (vanilla|adult) $52 / 93$ or .559
10. Find P (child|chocolate)
$105 / 146$ or .219
11. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

$$
22 / 45 \text { or } .4 \overline{8}
$$

