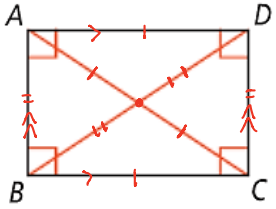
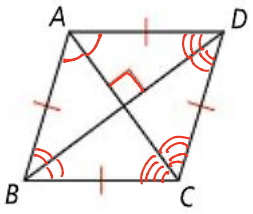
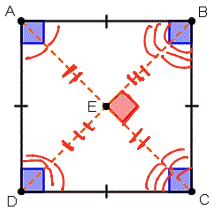
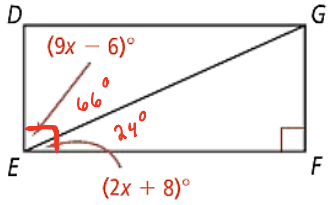
Math 3
Unit 6 Day 5 Notes – Quadrilaterals

Name: Key
Date: _____

| Rectangle | Rhombus | Square |
|---|---|---|
| A **rectangle** is a parallelogram with four right angles. | A **rhombus** is a parallelogram with four congruent sides. | A **square** is a parallelogram with four congruent sides and four right angles. |
| A **rectangle** has all the properties of a parallelogram PLUS:<br>• 4 right angles<br>• Diagonals are congruent $AC \cong BD$<br>  | A **rhombus** has all the properties of a parallelogram PLUS:<br>• 4 congruent sides<br>• Diagonals bisect angles<br>• Diagonals are perpendicular<br>  | A **square** has all the properties of a parallelogram PLUS:<br>• All the properties of a rectangle<br>• All the properties of a rhombus<br>$AC \cong BD$<br>  |

**Example 1:** Solve for x and the measure of each angle if ▭DGFE is a rectangle.




$9x-6+2x+8=90^\circ$

$11x+\cancel{2}=90^\circ$
$\cancel{2}\quad -2$

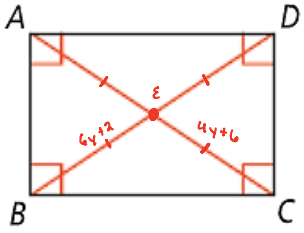
$\frac{11x}{11}=\frac{88}{11}$

$x=8$

$9(8)-6$
$m\angle DEG = 66^\circ$

$2(8)+8$
$m\angle GEF = 24^\circ$

**Example 2:** ▭ABCD is a rectangle whose diagonals intersect at point E.




a) If AE = 36 and CE = 2x – 4, find x.

$2x-\cancel{4}=36$
$\cancel{+4}\quad +4$

$\frac{2x}{2}=\frac{40}{2}$ $x=20$

b) If BE = 6y + 2 and CE = 4y + 6, find y.

$6y+\cancel{2}=4y+6$
$-4y\ \cancel{-2}\quad -4y\ -2$

$\frac{2y}{2}=\frac{4}{2}$

$y=2$

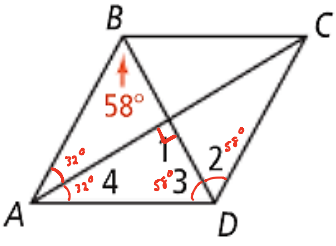
**Example 3:** Using the diagram to the right to answer the following if ▭ABCD is a rhombus.

a) Find the m∠1.
$=90^\circ$

b) Find the m∠2.
$=58^\circ$

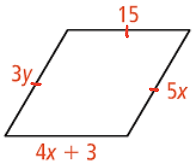
c) Find the m∠3.
$=58^\circ$

d) Find the m∠4.
$=32^\circ$




**Example 4:** Solve for each variable if the following are rhombi.

a)



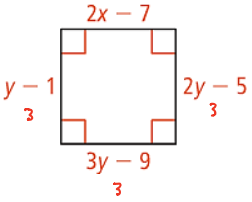

$\frac{5x}{5}=\frac{15}{5}$ $x=3$

$\frac{3y}{3}=\frac{15}{3}$ $y=5$

$4x+3=15$ $x=3$

$4x+3=5x$ $x=3$

b)




$3y-\cancel{9}=2y-5$
$-2y\ \cancel{+9}\quad -2y+9$

$y=4$

$2x-\cancel{7}=3$
$\cancel{+7}\quad +7$

$\frac{2x}{2}=\frac{10}{2}$

$x=5$

$y-1=2y-5$

$3y-9=y-1$

# Trapezoid

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides, called *bases*, and two nonparallel sides, called *legs*.

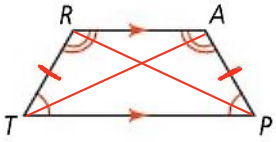
## Isosceles Trapezoids

An **isosceles trapezoid** is a trapezoid with congruent legs.

A trapezoid is isosceles if there is only:

- One set of parallel sides
- Base angles are congruent
- Legs are congruent $RT \cong AP$
- Diagonals are congruent $RP \cong TA$
- Opposite angles are supplementary $\angle T + \angle A = 180^\circ$, $\angle R + \angle P = 180^\circ$

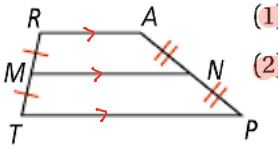
$\angle T \cong \angle P, \angle R \cong \angle A$




## Trapezoid Midsegment

The **median** (also called the midsegment) of a trapezoid is a segment that connects the midpoint of one leg to the midpoint of the other leg.

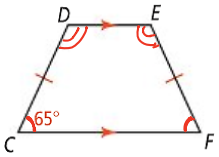
**Theorem:** If a quadrilateral is a trapezoid, then a) the midsegment is parallel to the bases and b) the length of the midsegment is half the sum of the lengths of the bases




(a) (1) $\overline{MN} \parallel \overline{TP}, \overline{MN} \parallel \overline{RA}$, and

(b) (2) $MN = \frac{1}{2}(TP + RA)$

**Example 5:** CDEP is an isosceles trapezoid and m<C = 65. What are m<D, m<E, and m<F?

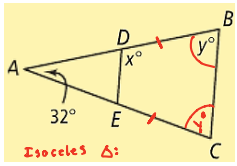



$m\angle D = 180^\circ - 65^\circ = 115^\circ$

$m\angle E = 115^\circ$

$m\angle F = 65^\circ$

**Example 6:** What are the values of x and y in the isosceles triangle below if DE || DC?




$AC \cong AB$ $\quad \angle B \cong \angle C$

$y + y + 32 = 180^\circ$

$2y + 32 = 180^\circ$

$-32 \quad -32$

$\frac{2y}{2} = \frac{148^\circ}{2}$

$y = 74^\circ$

$x^\circ = 180 - 74 = 106^\circ$

**Example 7:** QR is the midsegment of trapezoid LMNP. What is x and the length of LM?

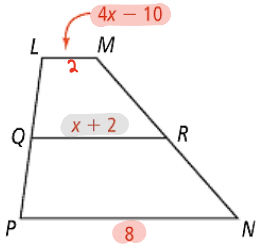
$\frac{1}{2}(4x - 10 + 8) = (x + 2)\frac{1}{2}$

$4x - 2 = 2x + 4$

$-2x \quad -2x + 2$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$ $\quad LM = 2$




**You Try!** TU is the midsegment of trapezoid WXYZ. What is x and the length of TU?

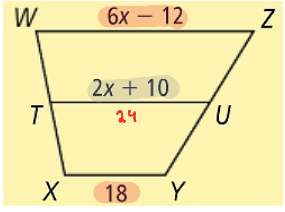
$\frac{1}{2}(6x - 12 + 18) = (2x + 10)\frac{1}{2}$

$6x + 6 = 4x + 20$

$-4x \quad -6 \quad -4x \quad -6$

$\frac{2x}{2} = \frac{14}{2}$

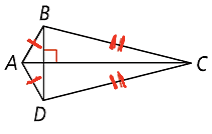
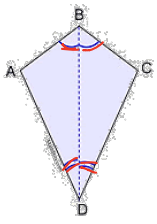
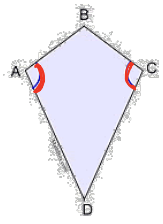
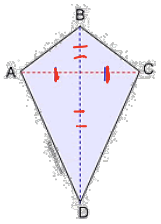
$x = 7$ $\quad TU = 24$




## Kite

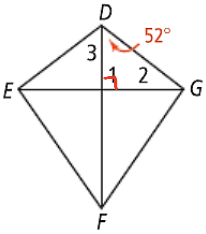
A **kite** is a quadrilateral with two pairs of adjacent, congruent sides.

If a quadrilateral is a kite, then:

| Its diagonals are perpendicular. | Its diagonals bisect the opposite angles. | One pair of opposite angles are congruent. | One diagonal bisects the other. |
|---|---|---|---|
| $\overline{AC} \perp \overline{BD}$  |  |  |  |



**Example 4:** Quadrilateral DEFG is a kite. What are m<1, m<2, and m<3?

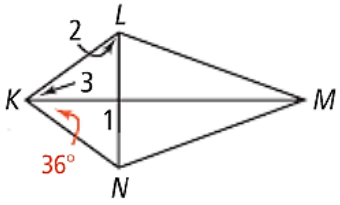



$m\angle 1 = 90^\circ$

$m\angle 2 = 90^\circ - 52^\circ = 38^\circ$

$m\angle 3 = 52^\circ$

**You Try!** Quadrilateral KLMN is a kite. What are m<1, m<2, and m<3?




$m\angle 1 = 90^\circ$

$m\angle 2 = 90 - 36^\circ = 54^\circ$

$m\angle 3 = 36^\circ$