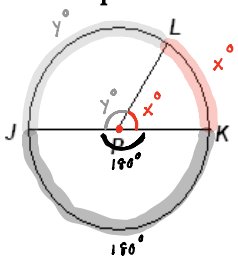
Math 3 Name: Key

Unit 6 Day 8 Notes – Arcs & Angles Date:_____

Part 1: Central Angles

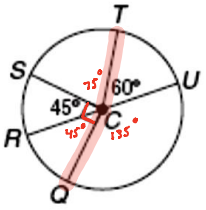
A central angle is an angle whose vertex is the center of the circle and whose other two points lie on the circle. $\angle LPK$ and $\angle JPL$ are central angles in circle P.



Measure of a central angle = Measure of its intercepted arc

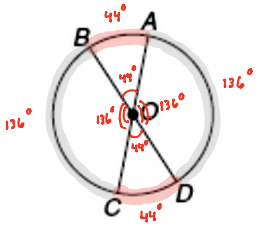
Find each measure.

1. $m\angle SCT$ = 75°
2. $m\angle SCU$ = 135°
3. $m\angle SCQ$ = 90°
4. $m\angle QCT$ = 195° or 165°



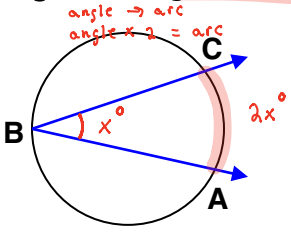
In $\odot O$, $m\angle BOA = 44$. Find each measure.

5. $m\widehat{BA}$ = 44°
6. $m\widehat{BC}$ = 136°
7. $m\widehat{CD}$ = 44°
8. $m\widehat{ACB}$ = 316°
9. $m\widehat{BCD}$ = 180°
10. $m\widehat{AD}$ = 136°



Part 2: Inscribed Angles

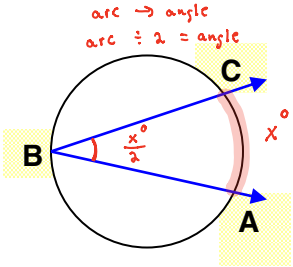
An inscribed angle is an angle whose vertex is ON the circle and whose sides contain chords of the circle.



$\angle CBA$ is an inscribed angle.

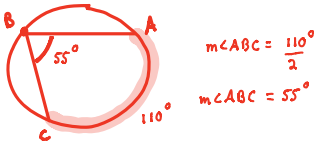
Minor arc CA is the intercepted arc of $\angle CBA$

The measure of an inscribed angle is half the measure of the intercepted arc.



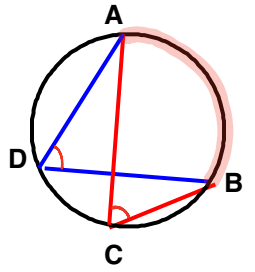
$$m\angle CBA = \frac{1}{2} m(arc\ CA)$$

For example: If the measure of arc CA is 110º, then $m\angle CBA = 55º$.



There are 3 corollaries that give us more information on the relationship between an inscribed angle and a circle.

Corollary 1: Two inscribed angles that intercept the same arc are congruent.



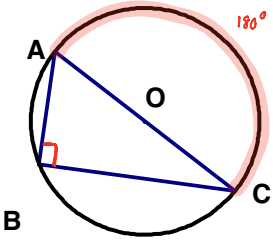
$m\angle ADB = m\angle ACB$

$\angle ADB \cong \angle ACB$

$\angle ADB$ intercepts arc AB; $\angle ACB$ intercepts arc AB

Therefore, $\angle ADB \cong \angle ACB$

Corollary 2: An angle inscribed in a semicircle is a right angle.



$m\angle ABC = 90°$

If $\overline{AC}$ is a diameter of Circle O, then $\angle ABC$ is a right angle.

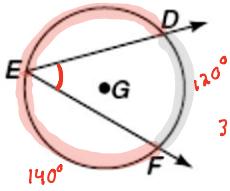
Corollary 3: The opposite angles of a quadrilateral inscribed in a circle are supplementary. (Remember an inscribed polygon has every vertex of the polygon touching the circle.)

11. Use the given circle for both problems.

a. If the $m\angle DEF = 70$, find the measure of arc DF.

$\overset{\frown}{DF} = 70 \times 2 = 140°$

$\overset{\frown}{DF} = 140°$



b. If the measure of arc DE = 100 and the measure of arc EF = 140, find the $m\angle DEF$

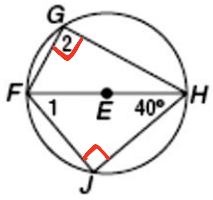
$360° - 140 - 100 = 120°$

$m\angle DEF = \frac{120}{2}$

$m\angle DEF = 60°$

12. Find the indicated angles.

$m\angle 1, m\angle 2$

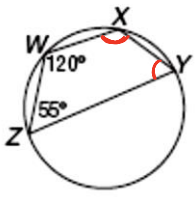


$m\angle 1 = 50°$

$m\angle 2 = 90°$

13. Find the indicated angles.

$m\angle X, m\angle Y$



$(180 - 55)$

$m\angle X = 125°$

$m\angle Y = 60°$

$(180 - 120)$

Now, "You Try" these:

14. Refer to the figure. Find each measure.

a. $m\angle ABC = 90°$

b. $m\overset{\frown}{CD} = 118°$

c. $m\overset{\frown}{AD} = 62°$

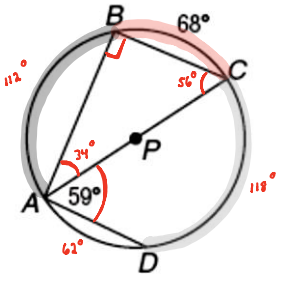
d. $m\angle BAC = 34°$

e. $m\angle BCA = 56°$

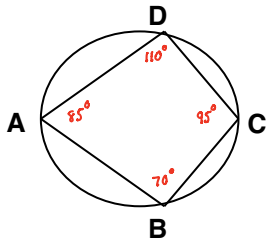
f. $m\overset{\frown}{AB} = 112°$

g. $m\overset{\frown}{BCD} = 186°$

h. $m\overset{\frown}{BDA} = 248°$

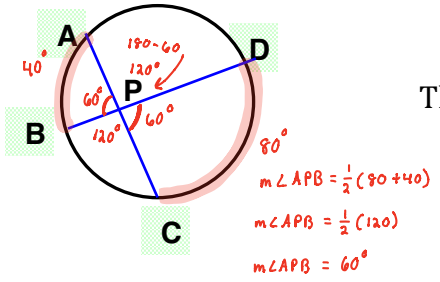


15. Find $m\angle D$ and $m\angle C$ if $m\angle A = 85$ and $m\angle B = 70$.



Part 3: Angles Formed Two Chords of a Circle (Vertex not at center of circle)

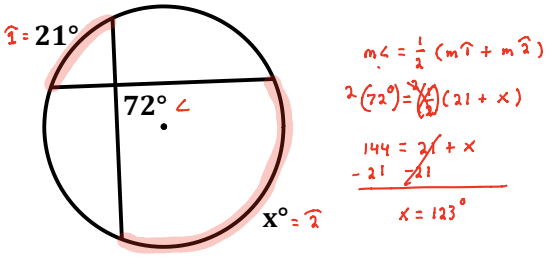
An angle formed by two chords of a circle whose vertex is not at the center of the circle has a measure equal to half the SUM of the intercepted arcs.



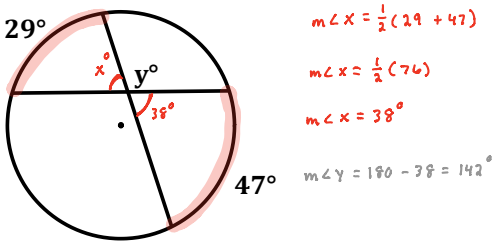
The $m\angle APB = \frac{1}{2}(m(arcAB) + m(arcDC))$

For example: If the measure of arc AB is 40º and the measure of arc DC is 80°, the $m\angle APB = 60°$.

16) Find the value of *x*.

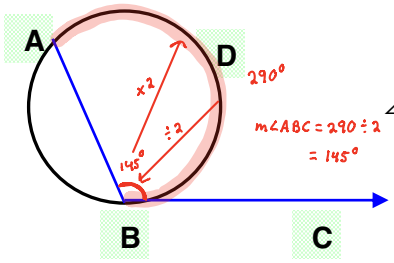


17) Find the value of *y*.



Part 4: Angles Formed By Secants, Tangents and/or Chords

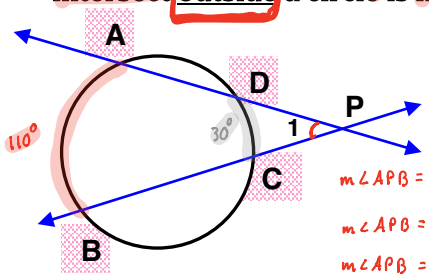
An angle formed by a tangent and a secant (or chord) that intersect on a circle at the point of tangency is one-half the measure of the intercepted arc.



$\angle ABC$ intercepts arc AB. The $m\angle ABC = \frac{1}{2}m(\text{arc } AB)$

For example: If the measure of arc ADB is 290º, then $m\angle ABC = 145º$.

The measure of an angle formed by two secants, a tangent and a secant, or two tangents that intersect outside a circle is half the DIFFERENCE of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(m(arcAB) - m(arcCD))$$

$m\angle APB = \frac{1}{2}(110^\circ - 30^\circ)$

$m\angle APB = \frac{1}{2}(80)$

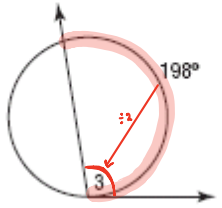
$m\angle APB = 40^\circ$

For example: If the measure of arc AB is 110 and the measure of arc CD is 30, then the

$$m\angle APB = \frac{1}{2}(110-30) = \frac{1}{2}(80) = 40^\circ.$$

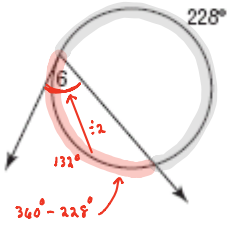
Find the indicated angle measure

18. $m\angle 3$



$m\angle 3 = 198 \div 2 = 99^\circ$

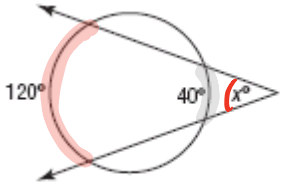
19. $m\angle 6$



$m\angle 6 = 132 \div 2 = 66^\circ$

Find the value of x.

20.

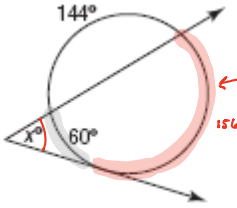


$m\angle x = \frac{1}{2}(120 - 40)$

$m\angle x = \frac{1}{2}(80)$

$m\angle x = 40^\circ$

21.

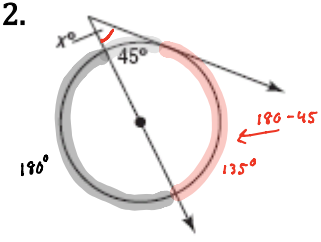


$m\angle x = \frac{1}{2}(156 - 60)$

$m\angle x = \frac{1}{2}(96)$

$m\angle x = 48^\circ$

22.

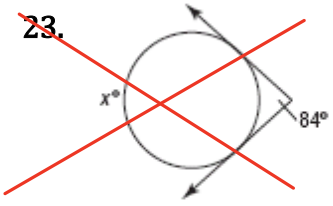


$m\angle x = \frac{1}{2}(135 - 45)$

$m\angle x = \frac{1}{2}(90)$

$m\angle x = 45^\circ$

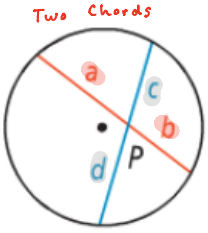
23.



Part 5: Segments Formed By Secants, Tangents and/or Chords

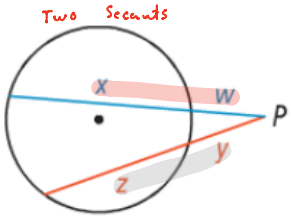
For a given point and circle, the PRODUCT of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.

Two Chords



$a \cdot b = c \cdot d$

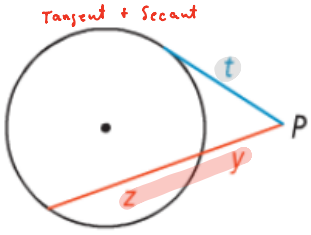
Two Secants



$(w + x)w = (y + z)y$

$w^2 + wx = y^2 + yz$

Tangent + Secant

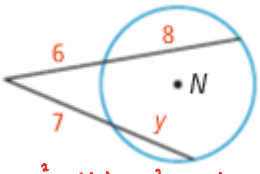


$(y + z)y = t^2$

$y^2 + yz = t^2$

Example 4: Find the value of the variable in $\odot O$.

a)



$6^2 + 8(6) = 7^2 + 7(y)$

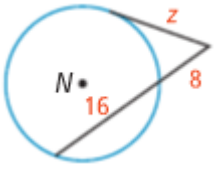
$84 = 49 + 7y$

$-49 \quad -49$

$\frac{35}{7} = \frac{7y}{7}$

$y = 5$

b)

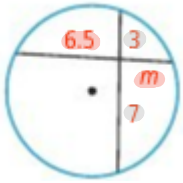


$z^2 = 8^2 + 8(16)$

$\sqrt{z^2} = \sqrt{192}$

$z = 13.9$

c)

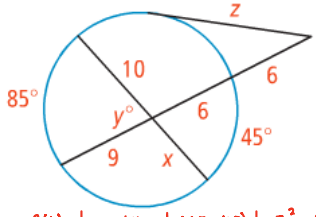


$6.5m = 3(7)$

$\frac{6.5m}{6.5} = \frac{21}{6.5}$

$m = 3.2$

You Try! What is the value of the variable to the nearest tenth?



$10x = 9(6)$

$\frac{10x}{10} = \frac{54}{10}$

$x = 5.4$

$m\angle y = \frac{1}{2}(85 + 45)$

$m\angle y = \frac{1}{2}(130)$

$m\angle y = 65°$

$z^2 = 6^2 + 15(6)$

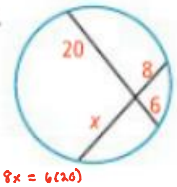
$z^2 = 36 + 90$

$\sqrt{z^2} = \sqrt{126}$

$z = 11.2$

Algebra Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth. See Problem 3.

15.

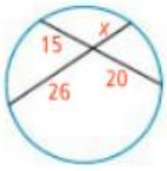


$8x = 6(20)$

$\frac{8x}{8} = \frac{120}{8}$

$x = 15$

16.

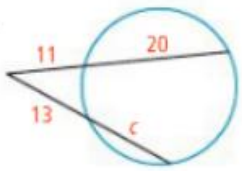


$26x = 15(20)$

$\frac{26x}{26} = \frac{300}{26}$

$x = 11.5$

17.



$13^2 + c(13) = 11^2 + 20(11)$

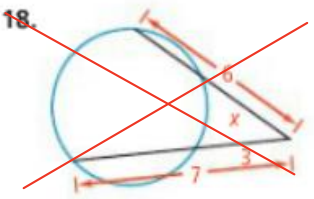
$169 + 13c = 341$

$-169 \quad -169$

$\frac{13c}{13} = \frac{172}{13}$

$c = 13.2$

18.



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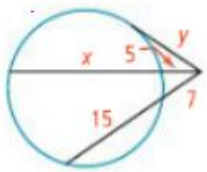


$5^2 + 5(x) = 7^2 + 7(15)$

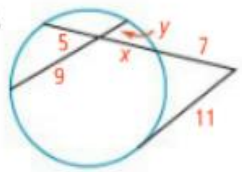
$25 + 5x = 154$

$-25 \quad -25$

$\frac{5x}{5} = \frac{129}{5}$

$x = 25.8$

20.



$7^2 + 7(x+5) = 11^2$

$49 + 7x + 35 = 121$

$7x + 84 = 121$

$-84 \quad -84$

$\frac{7x}{7} = \frac{37}{7}$

$x = 5.3$

$5y = 9(5.3)$

$\frac{5y}{5} = \frac{47.7}{5}$

$y = 9.54$