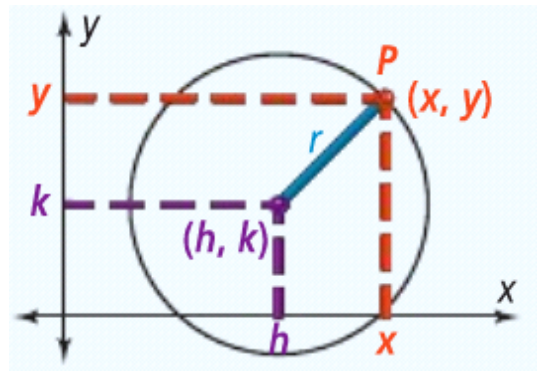


**Standard Form of Circles**

$C: (0,0)$ $x^2 + y^2 = r^2$	$C: (h,k)$ $(x-h)^2 + (y-k)^2 = r^2$
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<b>Center:</b> $(h, k)$	<b>Radius:</b> $r$	<b>Point on the circle:</b> $(x, y)$
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**Example 1:** Write the equation of a circle with the given information.

a) Center  $(0,0)$ , Radius=10

$h = 0$        $k = 0$        $r = 10^2 = 100$

$x^2 + y^2 = 100$

b) Center  $(2, 3)$ , Diameter=12  $\div 2$

$h = 2$        $k = 3$        $r = 6$

$(x-2)^2 + (y-3)^2 = 36$

**Example 2:** Determine the center and radius of a circle the given equation.

a)  $x^2 + y^2 = \frac{9}{4} = r^2$

$C: (0, 0)$      $r: \frac{3}{2}$

b)  $(x+3)^2 + (y-5)^2 = \sqrt{81} = r^2$

$C: (-3, 5)$      $r: 9$

c)  $(x+4)^2 + (y+6)^2 = \sqrt{1} = r^2$

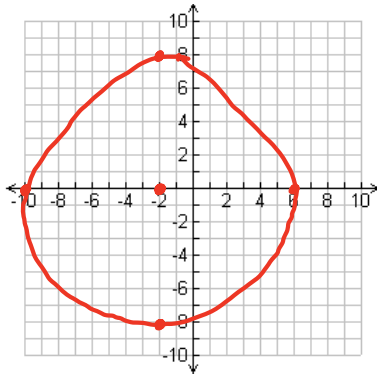
$C: (-4, -6)$      $r: 1$

**Example 3:** Use the center and the radius to graph each circle.

a)  $(x+2)^2 + y^2 = \sqrt{64}$

Center:  $(-2, 0)$

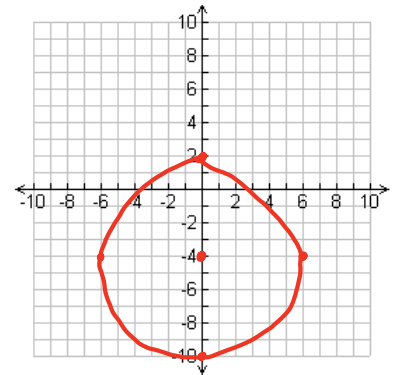
Radius: 8



b)  $x^2 + (y+4)^2 = 36$

Center:  $(0, -4)$

Radius: 6



### Writing an Equation with a Pass-Thru Point

**Step 1:** Substitute the center  $(h, k)$  into the equation

**Step 2:** Substitute the "pass through point  $(x, y)$ " into the equation for  $x$  and  $y$ .

**Step 3:** Simplify and solve for  $r^2$ .

**Step 4:** Substitute  $r^2$  back into the equation from Step 1.

**Example 4:** Write the equation of a circle with a given center  $(2, 5)$  that passes through the point  $(5, -1)$ .

$(x-h)^2 + (y-k)^2 = r^2$

$(5-2)^2 + (-1-5)^2 = r^2$

$(3)^2 + (-6)^2 = r^2$

$9 + 36 = r^2$

$45 = r^2$

$(x-2)^2 + (y-5)^2 = 45$

## Writing an Equation with Two Points on the Circle

## Midpoint Formula (center)

Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Example 5:** Write the equation of a circle with endpoints of diameter at  $(-6, 5)$  and  $(4, -3)$ .

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+1)^2 + (y-1)^2 = r^2$$

$$(-6+1)^2 + (5-1)^2 = r^2$$

$$(-5)^2 + (4)^2 = r^2$$

$$25 + 16 = r^2$$

$$41 = r^2$$

$$(x+1)^2 + (y-1)^2 = 41$$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Center:  $(-1, 1)$

## Writing the Equation of a Circle in Standard Form

<b>Step 1:</b>	Group <b>x's</b> and group <b>y's</b> together.
<b>Step 2:</b>	Move any constants to the right side of the equation.
<b>Step 3:</b>	Use <b>complete the square</b> to make a perfect square trinomial for the x's and then again for the y's. <i>*Remember, whatever you do to one side of the equation, you must do to the other!</i>
<b>Step 4:</b>	Simplify factors into standard form of a circle!

**Example 5:** Write the equation of a circle in standard form. Then, state the center and the radius.

a)  $x^2 + y^2 + 4x - 8y + 16 = 0$

b)  $x^2 + y^2 + 6x - 4y = 0$

$$x^2 + 4x + 4 + y^2 - 8y + 16 = -16 + 4 + 16$$

$$(x+2)^2 + (y-4)^2 = 4$$

$$C: (-2, 4) \quad r: 2$$

c)  $x^2 + y^2 - 6x - 2y + 4 = 0$

d)  $x^2 + y^2 + 8x - 10y - 4 = 0$

$$x^2 + 8x + 16 + y^2 - 10y + 25 = 4 + 16 + 25$$

$$(x+4)^2 + (y-5)^2 = 45$$

$$C: (-4, 5) \quad r = \sqrt{45} \text{ or } 3\sqrt{5}$$

$\left(\frac{4}{2}\right)^2$   
 $\downarrow$   
 $(2)^2$   
 $\downarrow$   
 $4$   
 $\oplus$   
 $\left(\frac{-8}{2}\right)^2$   
 $\downarrow$   
 $(-4)^2$   
 $\downarrow$   
 $16$

$\left(\frac{8}{2}\right)^2$   
 $\downarrow$   
 $(4)^2$   
 $\downarrow$   
 $16$   
 $\oplus$   
 $\left(\frac{-10}{2}\right)^2$   
 $\downarrow$   
 $(-5)^2$   
 $\downarrow$   
 $25$