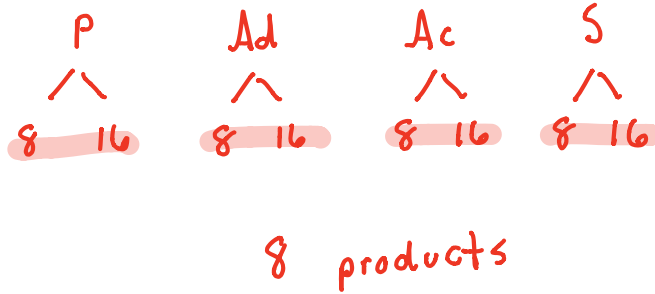


**Basic Counting Methods for Determining the Possible Outcomes**

A. Fundamental Counting Principle:

a. Tree Diagrams:

Example #1: Alpo manufactures 4 different types of dog food: puppy, adult, active and senior. Each type comes in two different sizes: 8 lbs or 16 lbs. Make a tree diagram representing the different products. How many different products can the company display?



b. In general:

- If there are  $m$  ways to make a first selection and  $n$  ways to make a second selection, then there are  $m \cdot n$  ways to make the two selections simultaneously. This is called the Fundamental Counting Principle.

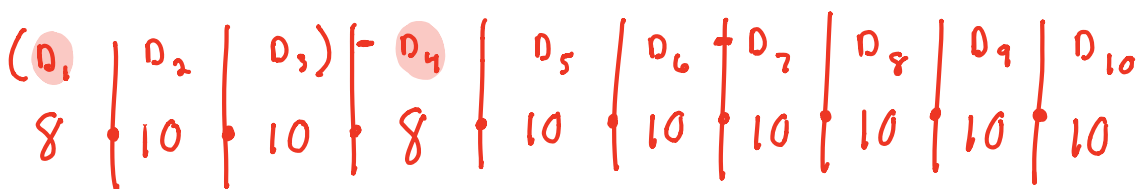
Example #1 above: 4 different types of dog food in 2 different sizes. How many different products?

$$4 \cdot 2 = 8 \text{ products}$$

Example #2: Joey has to pick out an outfit for school. He has 2 pairs of pants appropriate for school: one blue, one black. He has 3 shirts: one red, one green, one blue. He has 2 pairs of shoes to choose from: one blue, one black. How many different outfits can Joey select?

$$2 \cdot 3 \cdot 2 = 12 \text{ outfits}$$

Example #3 (more restricted) Telephone numbers, in US, begin with three-digit area codes followed by seven-digit local telephone numbers. How many different telephone numbers are possible? (Area codes and local telephone numbers cannot begin with 0 or 1)  $0-9 = 10 \text{ digits}$



$$6400000000$$

telephone #'s

Example #4 In a certain state, automobile license plates display 3 letters followed by 3 digits. How many such plates are possible if repetition of the letters is

(a) allowed?

$$\begin{array}{c}
 L_1 \mid L_2 \mid L_3 \mid D_1 \mid D_2 \mid D_3 \\
 26 \mid 26 \mid 26 \mid 10 \mid 10 \mid 10 \\
 \hline
 17576000 \text{ license plates}
 \end{array}$$

(b) not allowed?

$$\begin{array}{c}
 \overset{26}{\uparrow} \qquad \qquad \qquad \overset{10}{\uparrow} \\
 L_1 \mid L_2 \mid L_3 \mid D_1 \mid D_2 \mid D_3 \\
 26 \mid 25 \mid 24 \mid 10 \mid 9 \mid 8 \\
 \hline
 11232000 \text{ license plates}
 \end{array}$$

Example #5 In how many different ways can a race with six runners be completed? Assume there is no tie.

$$\begin{array}{c}
 1st \mid 2nd \mid 3rd \mid 4th \mid 5th \mid 6th \\
 6 \mid 5 \mid 4 \mid 3 \mid 2 \mid 1 \\
 \hline
 720 \text{ ways}
 \end{array}$$

Example #6 Three digit numbers are formed using the digits 2, 4, 5, and 7, with repetition of digits allowed. How many such numbers can be formed if

(a) the numbers are less than 700?

$$\begin{array}{c}
 D_1 \mid D_2 \mid D_3 \\
 3 \mid 4 \mid 4 = 48 \#s
 \end{array}$$

(b) the numbers are even?

$$\begin{array}{c}
 D_1 \mid D_2 \mid D_3 \\
 4 \mid 4 \mid 2 = 32 \#s
 \end{array}$$

(c) the numbers are divisible by 5?

$$\begin{array}{c}
 D_1 \mid D_2 \mid D_3 \\
 4 \mid 4 \mid 1 = 16 \#s
 \end{array}$$

(d) the number must start with a 2?

$$\begin{array}{c}
 D_1 \mid D_2 \mid D_3 \\
 1 \mid 4 \mid 4 = 16 \#s
 \end{array}$$

Example #7 A senate subcommittee consists of ten Democrats and seven Republicans. In how many ways can a chairman, vice chairman, and secretary be chosen if

a) there are no restrictions?

$$\begin{array}{c}
 C \mid VC \mid S \\
 17 \mid 16 \mid 15 = 4080 \text{ ways}
 \end{array}$$

b) the chairman must be a Democrat and the vice chairman must be a Republican?

$$\begin{array}{c}
 C \mid VC \mid S \\
 10 \mid 7 \mid 15 = 1050 \text{ ways}
 \end{array}$$

- 1) There are 6 people in a race. In how many ways can they finish first, second or third?
  
- 2) A golfer has 4 different hats, 3 gloves and 2 pairs of shoes to pick from for his round of golf. In how many ways can he make his choices?
  
- 3) In Canada, postal codes consist of 6 characters -- three letters and three digits. Each postal code starts with a letter and alternates with a digit.
  - a. How many postal codes are there?
  - b. How many start with the letter S?
  - c. How many start with the letter S and end in the digit 8?
  - d. How many start with the letter S, digit 6 and NO letter or digit is repeated?
  
- 4) Using the digits  $\{1, 2, 3, 4, 5\}$ , how many positive three digit integers can be made if:
  - a. there are NO restrictions
  - b. it is odd and repetition is allowed?
  - c. it is odd and repetition is NOT allowed?
  - d. Repeat question a, b and c if the digits you can choose are  $\{0, 1, 2, 3, 4, 5\}$ .
  
- 5) In how many ways can ALL of the letters of the word TRAVEL be arranged if:
  - a. there are NO restrictions?
  - b. it must start with T?
  - c. it starts with a consonant and ends in a vowel?
  - d. the letters TR must stay together?
  
- 6) How many positive even three-digit integers less than 400 can be formed from the digits  $\{0, 1, 2, 3, 4, 5\}$  if:
  - a. repetition is allowed?
  - b. No digit is repeated?

- 7) You are ordering dinner at a restaurant. How many ways can you order a meal if you have two choices for a drink ( coffee or tea ), three main courses to choose from ( chicken, beef, or fish ) and two desserts ( pie or cake ) ?
- Draw a tree diagram
  - Use the fundamental counting principle
- 8) Eight sprinters are in the final of a race. How many different ways there to award the gold, silver and bronze medals ?
- 9) Television stations in Canada usually have call letters that are 4 letters long and begin with the letter C. If the CRTC made this a law in Canada, then how many television stations could the CRTC license ?
- 10) Repeat the above question using the restriction, repetition of letters is NOT allowed
- 11) Some license plates consist of 3 letters followed by 3 numbers. How many different license plates are possible if:
- if there are NO Restrictions
  - if the letters must be DIFFERENT
  - if the letters are different and the first digit can't be 0
- 12) How many two digit whole numbers can be formed using the digits: 0,1,2,4,6,7,8,9 ( 8 digits ) ?
- Repetitions are allowed
  - Repetitions are not allowed
- 13) An ice cream parlor features 64 flavors and 20 toppings, in 3 sizes. How many different sundaes can be made ?



- 14) How many **EVEN** two digit numbers are there ?
- 15) How many **EVEN** two digit numbers can be made using the digits 1 , 2 , 3, 4, 5, 6, 7, 8 ?
- Repetitions are not allowed
  - Repetitions are allowed
- 16) How many two digit numbers can be formed using the digits 0 , 2 , 4 , 6 , 8 if:
- Repetitions are allowed
  - Repetitions are not allowed
- 17) How many **ODD** four digit numbers can be made from all of the digits, if
- Repetition is allowed
  - Repetition is not allowed
- 18) In how many ways can all of the letters of the word **PROBLEM** be arranged ?
- 19) In how many ways can all of the letters of the word **PROBLEM** be arranged if the arrangement must start with a consonant and end in a vowel ?
- 20) How many ways can the letters in the word **PENCIL** be arranged?
- 21) If there are four different types of cookies, how many ways can you eat all of them?
- 22) If three albums are placed in a multi-disc stereo, how many ways can the albums be played?
- 23) How many ways can you order the letters in **KEYBOARD** if **K** and **Y** must always be kept together?
- 24) How many ways can the letters in **OBTUSE** be ordered if all the vowels must be kept together?
- 25) How many ways can 4 rock, 5 pop, & 6 classical albums be ordered if all albums of the same genre must be kept together?