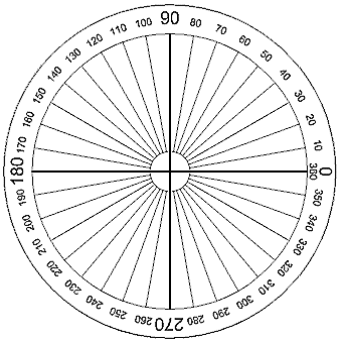
Math 3 Name Key

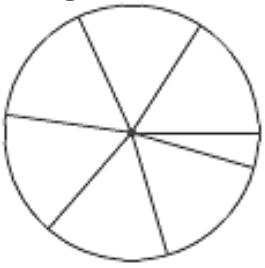
Unit 7 Day 1 Notes – Angle & Radian Measure Date _____



Units for Measuring Angles

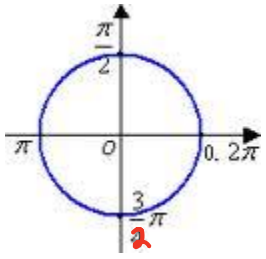
- **Degrees:** A circle is divided into 360 equal degrees, so that a right angle is $90°$

- **Radians:** One **radian** is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.



-The circumference of a circle with radius 1 is 2π so a complete revolution has made 2π radians (or approximately 6.28 radians as seen in the above figure).

-A straight angle (or 180^0 of a circle) has measure π radians.



Converting Radians and Degrees:

$D^\circ \times \frac{\pi}{180}$ don't use π when multiplying

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees} \quad (D^\circ \to R)$$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians} \quad (R \to D^\circ)$$

$R \times \frac{180}{\pi}$

Examples:

1. Express 60^o in radians $(D^\circ \to R)$

$D^\circ \times \frac{\pi}{180} \to 60^\circ \times \frac{\pi}{180} \to \frac{1}{3} \to \frac{1\pi}{3}$ or $\frac{\pi}{3}$

Type in Calc: $60 \div 180$

2. Express $\frac{\pi}{6}$ rad in degrees $(R \to D^\circ)$

$R \times \frac{180}{\pi}$

$\frac{\pi}{6} \times \frac{180}{\pi} \to \left(\frac{1}{6}\right) \times 180 \to 30^\circ$

On Your Own:

#1-8, change the given angle to radians.

1) 315° (D → R)

$315° \times \frac{\pi}{180} \rightarrow \frac{7\pi}{4}$

Type in Calc: $\frac{315}{180}$

2) -60°

$-\frac{\pi}{3}$

3) 212°

$\frac{53\pi}{45}$

4) -168°

$-\frac{14\pi}{15}$

5) 12.5°

$\frac{5\pi}{72}$

6) -310°

$-\frac{31\pi}{18}$

7) 600°

$\frac{10\pi}{3}$

8) -720°

-4π

#9-16, change the given angle to degrees.

9) $\frac{3\pi}{4}$ (R → D°)

$\frac{3\pi}{4} \times \frac{180}{\pi} \rightarrow \left(\frac{3}{4}\right) \times 180° \rightarrow 135°$

10) $-\frac{9\pi}{5}$

$-324°$

11) $\frac{15\pi}{8}$

$337.5°$

12) $-\frac{\pi}{10}$

$-18°$

13) $\frac{7\pi}{10}$

$126°$

14) $-\frac{16\pi}{15}$

$-192°$

15) $\frac{88\pi}{9}$

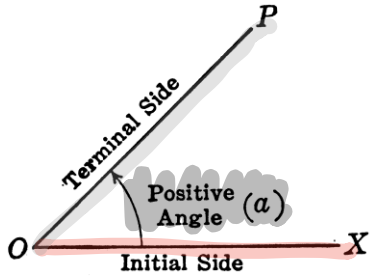
$1760°$

16) $-\frac{29\pi}{12}$

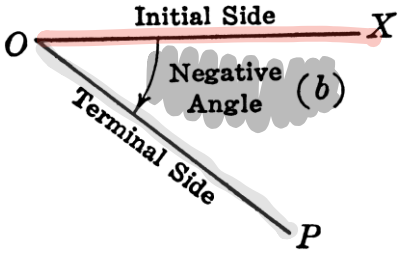
$-435°$

Angles in Standard Position

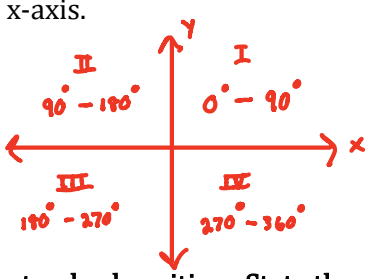
Angle: generated by the rotation of 2 rays that share a fixed endpoint (vertex)

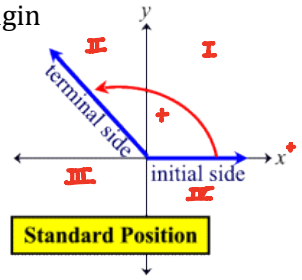


- **Initial Side:** fixed ray
- **Terminal Side:** ray that rotates away from initial side
- **Positive Angle:** counterclockwise rotation
- **Negative Angle:** clockwise rotation



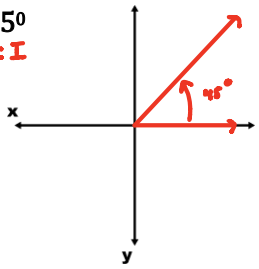
An angle is in **standard position** if it is draw in the xy-plane with its vertex at the origin and its initial side on the positive x-axis.



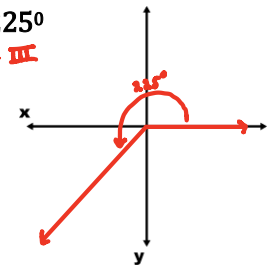


Example: Draw the given angle in standard position. State the quadrant in which the terminal side lies.

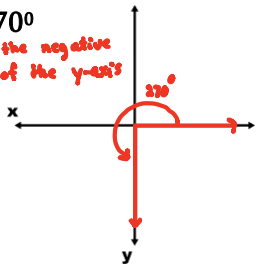
1. 45^0 Q: I



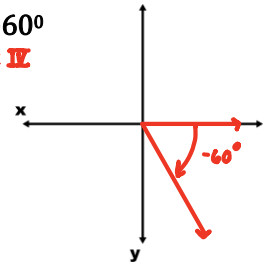
2. 225^0 Q: III



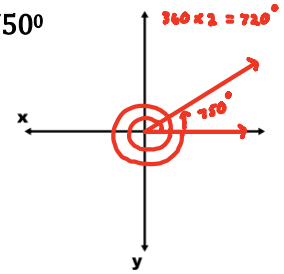
3. 270^0 on the negative part of the y-axis



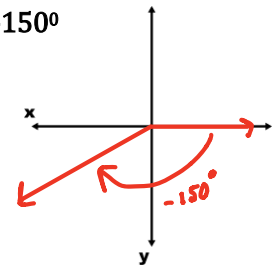
4. -60^0 Q: IV



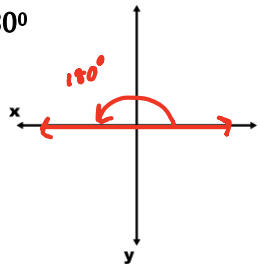
5. 750^0



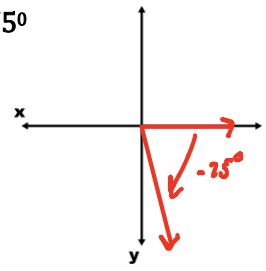
6. -150^0



7. 180^0

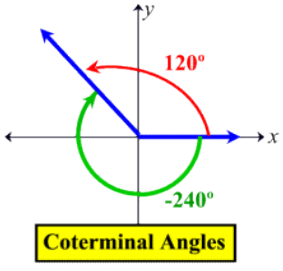


8. -75^0



Coterminal Angles

Two angles in standard position are **coterminal angles** if their terminal sides coincide. Every angle has infinitely many coterminal angles.



To find angles that are coterminal, add or subtract any multiple of 360° for degrees or 2π for radians.

Examples:

1. Find three angles that are coterminal with the angle $\theta = 30°$ in standard position

$30^\circ - 360^\circ = -330^\circ$ (5) $-360^\circ = -690^\circ$ (1) $-360^\circ = -1050^\circ$ (2)

$30^\circ + 360^\circ = 390^\circ$ (5) $+360^\circ = 750^\circ$ (1) $+360^\circ = 1110^\circ$ (2)

2. Find three angles that are coterminal with the angle $\theta = \frac{\pi}{3}$ in standard position

$\frac{\pi}{3} + \frac{2\pi \cdot 3}{2 \cdot 3} \rightarrow \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$ (2) $+ \frac{6\pi}{3} = \frac{14\pi}{3}$ (2) $+ \frac{6\pi}{3} = \frac{20\pi}{3}$ (3)

3. Find an angle with a measure between 0^o and 360^o that is coterminal with the angle of measure 1290^o in standard position.

$1290^\circ - 360^\circ = 930^\circ - 360^\circ = 570^\circ - 360^\circ = 210^\circ$

$-1290 + 360 = -930 + 360^\circ = -570^\circ + 360^\circ = -210 + 360 = 150^\circ$

"Why were the screams coming from the kitchen?"

Make the conversion from degrees to radians and from radians to degrees. To figure out the joke, place the letter of each problem above the answer on the line(s) below.

Convert #1-6 from degrees to radians.

E. 20° $\frac{\pi}{9}$

A. 120° $\frac{2\pi}{3}$

K. -145° $-\frac{29\pi}{36}$

H. 320° $\frac{16\pi}{9}$

G. -245° $-\frac{49\pi}{36}$

B. 630° $\frac{7\pi}{2}$

Convert #7-13 from radians to degrees.

N. $\frac{\pi}{4}$ 45°

W. $\frac{2\pi}{5}$ 72°

T. $\frac{-5\pi}{60}$ -15°

S. $\frac{7\pi}{6}$ 210°

C. $\frac{13\pi}{3}$ 780°

I. $\frac{-5\pi}{4}$ -225°

O. $\frac{11\pi}{3}$ 660°

T	H	E	C	O	O	K	W	A	S
-15°	$\frac{16\pi}{9}$	$\frac{\pi}{9}$	780°	660°	660°	$\frac{-29\pi}{36}$	72°	$\frac{2\pi}{3}$	210°

B	E	A	T	I	N	G
$\frac{7\pi}{2}$	$\frac{\pi}{9}$	$\frac{2\pi}{3}$	-15°	-225°	45°	$\frac{-49\pi}{36}$

T	H	E	E	G	G	S
-15°	$\frac{16\pi}{9}$	$\frac{\pi}{9}$	$\frac{\pi}{9}$	$\frac{-49\pi}{36}$	$\frac{-49\pi}{36}$	210°

Find a coterminal angle between 0° and 360°.

13) $-330°$

$30°$

14) $-435°$

$285°$

15) $640°$

$280°$

16) $-442°$

$278°$

Find a coterminal angle between 0 and 2π for each given angle.

17) $\frac{11\pi}{3}$ $-\frac{6\pi}{3}$

$\frac{5\pi}{3}$

18) $-\frac{35\pi}{18}$ $+\frac{36\pi}{18}$

$\frac{\pi}{18}$

19) $\frac{15\pi}{4}$ $-\frac{8\pi}{4}$

$\frac{7\pi}{4}$

20) $-\frac{19\pi}{12}$ $+\frac{24\pi}{12}$

$\frac{5\pi}{12}$

Find a positive and a negative coterminal angle for each given angle.

21) $\frac{5\pi}{4}$ $\pm\frac{8\pi}{4}$

$\frac{13\pi}{4}$ | $\frac{-3\pi}{4}$

22) $\frac{25\pi}{36}$ $\pm\frac{72\pi}{36}$

$\frac{97\pi}{36}$ | $\frac{-47\pi}{36}$

Warm Up: Unit 7 Day 1 Review **Name:** _____

Convert from degrees to radians.

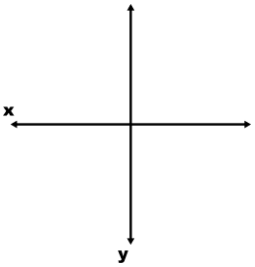
1. 212°
2. -85°
3. 415°

Convert from radians to degrees

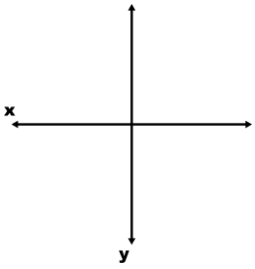
4. $-\frac{3\pi}{4}$
5. $\frac{15\pi}{19}$
6. $\frac{6\pi}{7}$

Draw the angle in standard position and state which quadrant the terminal side is in.

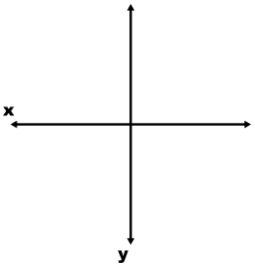
7. 275°



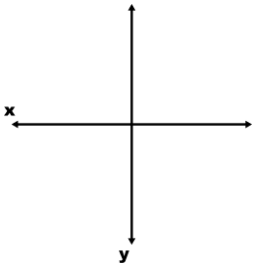
8. -105°



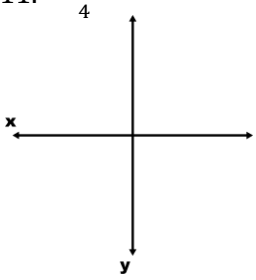
9. 480°



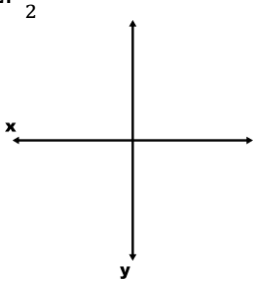
10. 170°



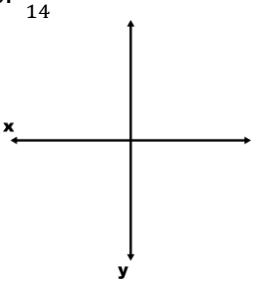
11. $-\frac{3\pi}{4}$



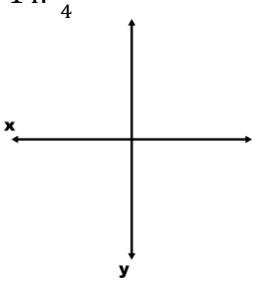
12. $\frac{9\pi}{2}$



13. $\frac{13\pi}{14}$



14. $\frac{7\pi}{4}$



Find a coterminal angle between 0° and 360°

15. 1770°
16. -412°

Find a coterminal angle between 0 and 2**π**

17. $-\frac{15\pi}{4}$
18. $\frac{27\pi}{5}$