AFM
Unit 7 Day 2 Notes – Permutations & Combinations

Name Key
Date _____

A ***permutation*** is an arrangement of a group of objects in a particular order. ORDER MATTERS!

The four numbers 5, 6, 7, and 8 can be arranged in twenty-four different ways:

5,6,7,8	5,6,8,7	5,7,6,8	5,7,8,6	5,8,6,7	5,8,7,6
6,7,8,5	6,7,5,8	6,8,5,7	6,8,7,5	6,5,7,8	6,5,8,7
7,5,6,8	7,5,8,6	7,6,5,8	7,6,8,5	7,8,5,6	7,8,6,5
8,5,6,7	8,5,7,6	8,6,5,7	8,6,7,5	8,7,5,6	8,7,6,5

Each of these arrangements is a different permutation. To determine the total number of permutations that can be made from four digits using each one only once, we indicate a space for each digit:

$4! = \_4\_ \cdot \_3\_ \cdot \_2\_ \cdot \_1\_ = 24$

You can choose any one of the four digits to put in the first space. Then you have any of the remaining three digits to go into the second space. There are now two choices for the third space, and the last digit is placed in the fourth space. The **product** of the number of choices for each space is the total number of permutations that can be made. So we have 24 different ways to fill the spaces.

Remember: Factorial (!) is defined as the product of each subsequent number until you reach 1

9! $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $= 362880$ 7! $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $= 5040$

Example: If we were arranging 6 objects we would have $6! = 720$ different ways to fill the spaces.

Any arrangement of n objects in a particular order is called a permutation of n objects.

We see that there are 24 permutations of 4 different objects and there are 720 permutations of 6 objects. The total number of permutations that can be formed from n objects using all of them without repetition is $n!$ The symbol $n!$ is read *n factorial*. Notice that ${}_nPn = n!$ because we are arranging **all** n objects.

What if you don't arrange all the items? can't be factorial!
You are given a set of 6 students who are members of a school club. From these 6 students you have to choose a president, vice-president, secretary and treasurer for the club. In how many ways can you do this? *(Notice that it makes a difference in the way that we choose them. We could choose the same four students but they could serve in different offices.)*

The symbol ${}_nP_k$ represents the number of permutations that can be formed from n objects taken k at a time where $k<n$. The number of permutations of six objects taken four at a time is

$$ {}_nP_k = \frac{n!}{(n-k)!} \qquad {}_6P_4 $$

Calculator: MATH → PRB → Opt. #2 ${}_nP_r$ (same as k)

Examples:

1. In a scrabble game, Mario drew the letters E,W,L,N,S,F and O. How many permutations of 4 letters are possible?

$_7P_4 = 840$

2. How many permutations are possible of the letters ABCDWXYZ?

$_8P_8 = 8! = 40320$

3. How many ways can a president, vice president, secretary and treasurer be elected from a club with 15 members?

$_{15}P_4 = 32760$

Permutations of objects that are <u>not</u> all different (Distinguishable Permutations)

The four numbers 1, 6, 6, 3 can be arranged in 4! ways. However, two of the numbers are the same so several of the arrangements are identical and cannot be **<u>distinguished</u>** from others. Consider the digits in this way: 1, 6, 6*, 3. The following arrangements can be made:

not distinguishable

16*63	16*36	166*3	1636*	136*6	1366*
6*631	6*613	6*136	6*163	6*316	6*361
516*3	6136*	6316*	636*1	66*13	66*31
316*6	3166*	36*16	36*61	3616*	366*1

Although we have written down twenty-four permutations, there are only twelve distinct arrangements. The number of distinct permutations of four objects when two are alike may be denoted by:

The number of distinct permutations of *n* objects of which *p* are alike, *q* are alike, *etc.* is: $\frac{n!}{(p!\,q!\,\dots)}$

Example: How many different permutations can be made using all the letters of the word Connecticut?

11

3 c's, 2 n's, 2 t's

$\frac{11!}{(3!\,2!\,2!)} = 1663200$

Example #2: Find the number of different ways of placing 15 balls in a row given that 4 are red, 3 are yellow, 6 are blue and 2 are black.

$$\frac{15!}{(4!\ 3!\ 6!\ 2!)}$$

$$= 6306300$$

Combinations

A ***combination*** is an arrangement of a group of objects in which ORDER DOES NOT MATTER !

From the numbers 1, 2, 3, six different permutations can be formed. They are 123, 132, 231, 213, 312, 321. When the order of the digits is not considered, **all six of these permutations make up one combination.** The number of combinations of three objects, taken three at a time is one. In general, the number of combinations of *n* objects taken *n* at a time is one.

Now, what if we want to look at the number of combinations that can be made from *n* objects, taking only *r* at a time is

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Calculator: MATH → PRB → Opt. # 3

Examples:

1. In how many ways can a committee of four be chosen from ten people?

$${}_{10}C_4 = 210$$

2. A student must answer 7 of the 10 questions on an exam. In how many ways can he choose the 7 questions?

$${}_{10}C_7 = 120$$

3. A committee composed of 3 math majors and 4 science majors is to be selected from a group of 20 math majors and 16 science majors. How many different committees can be formed?

$$\underset{\text{Math}}{\left({}_{20}C_3\right)} \cdot \underset{\text{Science}}{\left({}_{16}C_4\right)} = 2074800$$

Permutations and Combinations

Example: A committee of seven, consisting of a chairperson, vice-chairperson, secretary (P) and 4 other members (C), is to be chosen from a class of 20 students. In how many ways can the committee be chosen?

$$\left({}_{20}P_3\right) \cdot \left({}_{17}C_4\right) = 16279200$$

Determine if the following are permutations or combinations:

1. Creating an access code for a computer site using any 8 alphabet letters. C

2. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from. P

3. Voting to allow 10 new members to join a club when there are 25 that would like to join. C

4. Finding different ways to arrange a line-up for batters on a baseball team. P

5. Choosing 3 toppings for a pizza if there are 9 choices. C

Solve:

6. Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

$_7P_3 = 210$

7. A coach must choose how to select his five starters from a team of 12 players. How many different ways can the coach choose the starters?

$_{12}C_5 = 792$

8. John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

$_5C_2 = 10$

9. There are 25 people who work in an office together. Five of these people are selected to attend five different conferences. The first person selected will go to a conference in Hawaii, the second will go to New York, the third will go to San Diego, the fourth will go to Atlanta, and the fifth will go to Nashville. How many such selections are possible?

$_{25}P_5 = 6375600$

10. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names are drawn for first prize, second prize, and third prize?

$_{112}P_3 = 1367520$

11. A disc jockey has to choose three songs for the last few minutes of his evening show. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?

P

$_9P_3 = 504$

12. There are 25 people who work in an office together. Five of these people are selected to go together to the same conference in Orlando, Florida. How many ways can they choose this team of five people to go to the conference?

C

$_{25}C_5 = 53130$

Unit 7 Day 2 HW

1. How many distinguishable arrangements are possible using the letters from the word POTATO?

2. How many ways can five people line up at a checkout counter in a supermarket?

3. A company car that has a seating capacity of six is to be used by six employees who have formed a car pool. If only four of these employees can drive, how many possible seating arrangements are there for the group?

4. Find the number of distinguishable permutations of the letters SWEET.

5. How many different four-letter permutations are there for the letters in the word "minimum"?

6. A political science professor must select 4 students from her class of 12 students for a field trip to state legislature. In how many ways can she do it?

7. The Canyon Crest Academy basketball team is in a tournament with 5 other teams. In how many ways can the teams finish the tournament?

8. A baseball team has nine players. How many different batting orders are possible assuming that every play will be allowed to bat?

9. Find the number of different ways of placing 15 balls in a row given that 4 are red, 3 are yellow, 6 are black, & 2 are blue.

10. Fourteen construction workers are to be assigned to three different tasks. Seven workers are needed for mixing cement, five for laying bricks, & two for carrying bricks. In how many different ways can the workers be assigned to these tasks

11. If the NCAA has applications from 6 universities for hosting its intercollegiate tennis championships in 2008 and 2009, how many ways may they select the hosts for these championships
    a) if they are not both to be held at the same university?

    b) if they may both be held at the same university?

12. There are five finalists in the Mr. Rock Hill pageant. In how many ways may the judges choose a winner and a first runner-up?

13. In a primary election, there are four candidates for mayor, five candidates for city treasurer, and two candidates for county attorney. In how many ways may voters mark their ballots if they vote in all three of the races?

14. In how many ways may can five persons line up to get on a bus?

15. How many permutations are there of the letters in the word “great”?

16. How many distinct permutations are there of the word “statistics”?

17. How many distinct permutations of the word “statistics” begin and end with the letter “s”?

18. A college football team plays 10 games during the season. In how many ways can it end the season with 5 wins, 4 losses, and 1 tie?

19. If eight people eat dinner together, in how many different ways may 3 order chicken, 4 order steak, and 1 order lobster?

20. Suppose a True-False test has 20 questions.
    a) In how many ways may a student mark the test, if each question is answered?

    b) In how many ways may a student mark the test, if 7 questions are marked correctly and 13 incorrectly?

    c) In how many ways may a student mark the test, if 10 questions are marked correctly and 10 incorrectly?

21. Among the seven nominees for two vacancies on the city council are three men and four women. In how many ways may these vacancies be filled
    a) with any two of the nominees?

    b) with any two of the women?

    c) with one of the men and one of the women?

22. Mr. Jones owns 4 pairs of pants, 7 shirts, and 3 sweaters. In how many ways may he choose2 of the pairs of pants, 3 of the shirts, and 1 of the sweaters to pack for a trip?

23. In how many ways may one A, three B's, two C's, and one F be distributed among seven students in an AFM class?