AFM
Unit 7 Day 3 Notes – Theoretical vs. Experimental Probability

Name Key
Date _____

Definitions:

Experiment: *process that gives a definite result*	***Outcomes:*** *results*
Sample space: *set of all possible outcomes*	***Event:*** *subset of a sample space*

An experiment consists of rolling a die one time.

a) What is the sample space? 1, 2, 3, 4, 5, 6

b) What is the subset, "E", of an event showing an even number? 2, 4, 6

c) What is the probability of getting an even number?

$3/6 = 1/2$

To find a ***theoretical probability***, first list all possible outcomes. Then use the formula:
P(event) = number of favorable outcomes/total number of outcomes

Can also be stated as:

$P(\text{event}) = \frac{\text{what we want}}{\text{total possible outcomes}}$

Probability must fall between 0 and 1 inclusive:

$0 \le P(Event) \le 1$

never happens — definitely happens

If the value of P(Event) is:

Closer to 1: more likely it is to occur
Closer to 0: less likely it is to occur
Equal to 1: a "certain" event
Equal to 0: an "impossible" event

What is the probability I will get heads if I flip a coin? $1/2$ or .5 → 50%

You can collect data through observations or experiments and use the data to state the ***experimental probability***.

With your experiments:

Heads: 𝍸 II | Tails: 𝍸 𝍸 III (20)

Coin Toss	*P*(Heads)	*P*(Tails)
Experimental Probability	$7/20$	$13/20$
Theoretical Probability	$1/2$	$1/2$

Ask yourself some questions regarding the experiment. Were the results of each experiment what you expected? What would you expect if we were to repeat the process 1000 times? 10,000 times? 100,000 times? → Exp. prob. gets closer to Theoretical Prob.

Law of Large Numbers: As your number of outcomes reaches *very large numbers* your experimental probability will near the theoretical probability.

Most of the time we will use Theoretical Prob.
Experimental Prob. uses actual data that is collected.

Examples of Theoretical Probability: $2 \qquad 3 \rightarrow 2^3 = 8$

1. An experiment consists of tossing a coin three times and recording the results in order.
   a) What is the sample space?

   HHH HTH TTT
   HHT THT TTH
   HTT THH

   b) What is the subset, E, of an event showing "exactly two heads"?

   HHT
   THH
   HTH

   c) What is the subset, F, of an event showing "at least two heads"?

   HHH THH
   HHT HTH

2. Use same experiment from #1.
   a) What is the probability of getting exactly two heads?

   $3/8$

   b) What is the probability of getting at least two heads?

   $4/8 = 1/2$

   c) What is the probability of getting no heads?

   $1/8$

3. A five-card poker hand is drawn from a deck of 52 cards. What is the probability that all five cards are spades? ↓ choosing more than one thing, must use COMBINATION
   a) What is the sample space?

   $_{52}C_5 = 2598960$

   b) How many ways can five spades be chosen?

   $_{13}C_5 = 1287$

   c) What is the probability that all five cards are spades?

   $$\frac{_{13}C_5}{_{52}C_5} = \frac{1287}{2598960} = \frac{4.952 \times 10^{-4}}{.0004952}$$

4. A bucket contains 10 red balls and 15 green balls. Six balls are drawn at random from the bucket. What is the probability that at least one ball is red?
   a) What is the sample space?

   $_{25}C_6 = 177100$

   b) What are the ways of choosing no red balls? (all green)

   $_{15}C_6 = 5005$

   c) Probability that at least one ball is red?

   $$177100 - 5005 = \frac{172095}{177100} = \frac{447}{460} \approx .97$$

Theoretical Prob.

5. Suppose you have a bag with 75 marbles: 15 red, 5 white, 25 green, 20 black, and 10 blue. You draw a marble, note its color, and then put it back. You do this 75 times with these results: 12 red, 9 white, 27 green, 17 black, and 10 blue. Write each probability as a fraction in simplest form.

Experimental Prob.

Drawing Marbles	1. *P*(red)	2. *P*(white)	3. *P*(green)	4. *P*(black)	5. *P*(blue)
Experimental Probability	$12/75 = 4/15$	$9/75 = 3/25$	$27/75 = 9/25$	$17/75$	
Theoretical Probability	$15/75 = 1/5$	$5/75 = 1/15$	$25/75 = 1/3$	$20/75 = 4/15$	$10/75 = 2/15$

Replacement/Non-Replacement

Think about this...

Suppose you have 2 bags with the exact same sets of marbles inside. Let's say there are 4 red, 5 blue and 9 yellow marbles in each bag.

From the first bag, you reach in and make a selection. You record the color and then drop the marble back into the bag. You then repeat the experiment a second time.

From the second bag you do exactly the same thing EXCEPT, after you select the first marble and record it's color, you do NOT put the marble back into the bag, You then select a second marble, just like the other experiment.

Multiple Events
"And" → Multiply
"Or" → Add

The first experiment involves a process called *"with replacement"*.

# of outcomes on second try did not change

The second experiment involves a process called *"without replacement"*.

# of outcomes decreased

As you might imagine, the probabilities for the 2 experiments will not be the same. In this lesson we will illustrate a variety of these types of problems and explain how to arrive at the correct solutions.

Examples:

1. A player is dealt 2 cards from a standard deck of 52 cards. What is the probability of getting a pair of aces? → not a multiple event, only dealt one hand

$$\frac{{}_4C_2}{{}_{52}C_2} = \frac{1}{221} \text{ or } .0045$$

2. A jar contains 2 red and 5 green marbles. A marble is drawn, it's color noted, and put back in the jar. This process is repeated a total of 4 times. What is the probability that you selected 4 green marbles? → multiple events

1st	2nd	3rd	4th
$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{7}$	$\frac{5}{7}$

$$= \left(\frac{5}{7}\right)^4 = \frac{625}{2401} \text{ or } .2603$$

3. A bag of candy contains 4 lemon flavored sour balls, and 5 lime flavored sour balls. If Tim reaches in, takes one out and eats it, and then 20 minutes later selects another and eats that one as well, what is the probability that they were both lemon flavored candies?

$\left(\frac{4}{9}\right) \cdot \left(\frac{3}{8}\right) = \frac{12}{72} = \frac{1}{6}$ or $.1\overline{6}$

4. Mary has 4 dimes, 3 quarters and 7 nickels in her purse. She reaches in and pulls out a coin, only to have it slip from her fingers and fall back into the purse. She then picks out another coin. What is the probability that she picked a nickel on both tries?

$\left(\frac{7}{14}\right) \cdot \left(\frac{7}{14}\right) = \left(\frac{7}{14}\right)^2 = \frac{49}{196} = \frac{1}{4}$ or $.25$

5. A bag of candy contains 4 lemon flavored sour balls, and 5 lime flavored sour balls. If Tim reaches in, takes one out and eats it, and then 20 minutes later selects another and eats that one as well, what is the probability that the first was lemon and the second was lime?

$\left(\frac{4}{9}\right) \cdot \left(\frac{5}{8}\right) = \frac{20}{72} = \frac{5}{18}$ or $.2\overline{7}$

Unit 7 Day 3 HW(1)

1. A letter is selected at random from the letters of the word FLORIDA. What is the probability that the letter is an A? The letter is a vowel?

2. Find the probability of getting a heart if you are dealt one card from a standard 52 card deck.

3. Find the probability of getting a number less than 5 when you roll a die once.

4. Eight horses are entered in a race. You randomly predict a particular order for the horses to complete the race. What is the probability that your prediction is correct?

5. A bag contains 20 tennis balls, of which four are defective. If two balls are selected at random from the bag, what is the probability that both are defective?

Unit 7 Day 3 HW(2) – Experimental and Theoretical Probability

1. An experiment consists of tossing a coin and rolling a die.
   (a) Find the sample space

   (b) Find the probability of getting heads and an even number.

   (c) Find the probability of getting heads and a number greater than 4.

   (d) Find the probability of getting tails and an odd number.

2. A card is drawn randomly from a standard 52-card deck. Find the probability of the given event:
   (a) The card is a heart

   (b) The card is a heart or a spade.

   (c) The card is a heart, spade or diamond.

3. A child's game consists of a spinner as shown in the figure. Find the probability of the given event.
   (a) The spinner stops on an even number.

   (b) The spinner stops on an odd number or a number greater than 3.

4. A poker hand, consisting of five cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains:
   (a) 5 hearts

   (b) 5 face cards

5. A pair of dice is rolled and the numbers showing are observed.
   (a) List the sample space of this experiment.

   (b) Find the probability of getting a sum of 7.

   (c) Find the probability of getting a sum of 9.

   (d) Find the probability that the two dice show doubles.

6. What is the probability that a 13-card bridge hand consists of all cards from the same suit?

7. A toddler has wooden blocks showing the letters C, E, F, H, N and R. Find the probability that the child arranges the letters in the indicated order.
   (a) In the order FRENCH

   (b) In alphabetical order

8. The president of a large company selects six employees to receive a special bonus. He claims that the six employees are chosen randomly from among the 30 employees, of which 19 are women and 11 are men. What is the probability that no woman is chosen?

9. Eight horses are entered in a race. You randomly predict a particular order for the horses to complete the race. What is the probability that your prediction is correct?

Unit 7 Day 3 HW(3)

1. A coin and a die are tossed. Calculate the probability of getting tails and a 5.

2. In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+?

3. If a baseball player gets a hit in 31% of his at-bats, what it the probability that the baseball player will get a hit in 5 at-bats in a row?

4. What is the probability of tossing 2 coins one after the other and getting 1 head and 1 tail?

5. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be clubs?

6. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?

7. If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of *not* receiving any mail for 3 days in a row?

8. Jonathan is rolling 2 dice and needs to roll an 11 to win the game he is playing. What is the probability that Jonathan wins the game?

9. Thomas bought a bag of jelly beans that contained 10 red jelly beans, 15 blue jelly beans, and 12 green jelly beans. What is the probability of Thomas reaching into the bag and pulling out a blue or green jelly bean and then reaching in again and pulling out a red jelly bean? Assume that the first jelly bean is not replaced.

10. For question 9, what if the order was reversed? In other words, what is the probability of Thomas reaching into the bag and pulling out a red jelly bean and then reaching in again and pulling out a blue or green jelly bean *without replacement*?

11. What is the probability of drawing 2 face cards one after the other from a standard deck of cards *without replacement*?

12. There are 3 quarters, 7 dimes, 13 nickels, and 27 pennies in Jonah's piggy bank. If Jonah chooses 2 of the coins at random one after the other, what is the probability that the first coin chosen is a nickel and the second coin chosen is a quarter? Assume that the first coin is not replaced.

13. Jenny bought a half-dozen doughnuts, and she plans to randomly select 1 doughnut each morning and eat it for breakfast until all the doughnuts are gone. If there are 3 glazed, 1 jelly, and 2 plain doughnuts, what is the probability that the last doughnut Jenny eats is a jelly doughnut?

14. Steve will draw 2 cards one after the other from a standard deck of cards *without replacement*. What is the probability that his 2 cards will consist of a heart and a diamond?