Math 3 Name Key

Unit 7 Day 4 Notes – Properties of Logarithms

Logarithms are Exponents

Remember that logarithms are exponents, so the properties of exponents are the properties of logarithms.

1. **Product Property**

What is the rule when you multiply two values with the same base together ($x^2 * x^3$)? The rule is that you keep the base and add the exponents. Well, remember that logarithms are exponents, and when you multiply, you're going to add the logarithms.

The log of a product is the sum of the logs.

Condense

$$\log_a xy = \log_a x + \log_a y$$

Expand

2. **Quotient Property**

The rule when you divide two values with the same base is to subtract the exponents. Therefore, the rule for division is to subtract the logarithms.

The log of a quotient is the difference of the logs.

Condense

$$\log_a (x/y) = \log_a x - \log_a y$$

Expand

3. **Power Property**

When you raise a quantity to a power, the rule is that you multiply the exponents together. In this case, one of the exponents will be the log, and the other exponent will be the power you're raising the quantity to.

The exponent on the argument is the coefficient of the log.

Condense

$$\log_a x^r = r * \log_a x$$

Expand

Additonal Rules:

4. $\log_b 1 = 0$ ($b^0 = 1$)
5. $\log_b b = 1$ ($b^1 = b$)
6. $\log_b b^2 = 2$
7. $\log_b b^x = x$

State the property or properties used to rewrite each expression.

1. $\log 6 = \log 2 + \log 3$ (product) Product

2. $\log_b \frac{x^2}{y} = 2\log_b x - \log_b y$ (power, Quotient) Power & Quotient

3. $3\log_b 4 - 3\log_b 2 = \log_b 8$ (power, Quotient) Power & Quotient

4. $2\log_2 m - 4\log_2 n = \log_2 \frac{m^2}{n^4}$ Power & Quotient

Simplify. Write each expression as a single logarithm or term.

5. $\log_4 64 - \log_4 16$ $\log_4 \frac{64}{16} = \log_4 4 = 1$

6. $6\log_2 x + \log_2 y$ $\log_2 x^6 + \log_2 y = \log_2 x^6 y$

7. $3\log 2 + \log 4 - \log 16$ $\log 2^3 + \log 4 - \log 16$ $\log \frac{2^3 \cdot 4}{16} = \log \frac{8 \cdot 4}{16} = \log \frac{32}{16} = \log 2$

Can you write $3\log_2 9 - \log_6 9$ as a single logarithm?

No, they don't have the same base.

Expand each logarithm.

8. $\log_7 \frac{t}{u}$ $\log_7 t - \log_7 u$

9. $\log(4p^3)$ $\log 4 + \log p^3$ $\log 4 + 3\log p$

10. $\log\left(\frac{y}{3}\right)^2$ $2(\log y - \log 3)$

11. $\log_7 a^3 b^4$ $\log_7 a^3 + \log_7 b^4$ $3\log_7 a + 4\log_7 b$

$\frac{\sqrt[2]{x^1} = x^{\frac{1}{2}}}{\sqrt[3]{x^1} = x^{\frac{1}{3}}}$

12. $\log\sqrt{\frac{2x}{y}} = \log\left(\frac{2x}{y}\right)^{\frac{1}{2}}$ $\frac{1}{2}(\log 2x - \log y)$

Evaluate.

13. $\log_2 4 - \log_2 16$ $\log_2 \frac{4}{16} = \log_2 \frac{1}{4} = \log_2 \frac{1}{2^2} = \log_2 2^{-2} = -2$

14. $2\log_3 3 - \log_3 3$ $\log_3 3^2 - \log_3 3^1$ $2 - 1 = 1$