AFM **Name** Key

Unit 7 Day 5 Notes – Expected Value and Fairness **Date** ____________

<u>**Expected Value:**</u> the *weighted average* of possible values of a random variable, with weights given by their respective theoretical probabilities.

- The expected value informs about what to expect in an experiment "in the long run", after many trials.

A game gives payoffs of $a_1, a_2, a_3, \ldots, a_n$ with the probabilities $p_1, p_2, p_3, \ldots, p_n$. The expected value (or expectation) E of this game is: $E = a_1p_1 + a_2p_2 + a_3p_3 + \ldots + a_np_n$

Example:

When you roll a die, you will be paid $1 for odd number and $2 for even number. Find the expected value of money you get for one roll of the die.

The sample space of the experiment is {1, 2, 3, 4, 5, 6}.

The table illustrates the probability distribution for a single roll of a die and the amount that will be paid for each outcome.

Roll (X)	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Amount($)	1	2	1	2	1	2

Use the weighted average formula.

$$E(x) = \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right)$$
$$= \frac{1}{6} + \frac{2}{6} + \frac{1}{6} + \frac{2}{6} + \frac{1}{6} + \frac{2}{6}$$
$$= \frac{9}{6}$$
$$= 1.5$$

So, the expected value is $1.50. In other words, on average, you get $1.50 per roll.

NOTE! Expected value can be negative! A negative expected value indicates a negative payout (i.e. you're losing money!)

Ex 1: A coin is flipped. Heads, you win $1. Tails, you lose $1. What is the expected value of this game?

<u>A game whose expected winnings are $0 is called a **fair game.**</u>

	Prob.	$
Heads	1/2	+$1
Tails	1/2	-$1

$(\frac{1}{2})(1) + (\frac{1}{2})(-1)$

EV = $0

Fairness: Occurs when the probability of winning is *equally as likely* (meaning you have the same chance of winning and losing) or when expected value is such that a player can "break even" (meaning that after playing a game numerous times, his returns will match what he pays to play the game, E=0)

Ex 2: Jane gets \$6 if a die shows a 6 and loses \$1 otherwise. What is her expectation?

	Prob	\$
Roll 6	1/6	+\$6
Any other #	5/6	-\$1

$(1/6)(6) + (5/6)(-1)$

$= .17$ ¢ / roll

Ex 3: A die is rolled. If the die shows a 1, 2, or 3 you get 10 points. If the die shows a 4 or a 5, you lose 13 points. If the die shows a 6, you lose 1 point. What is the expected value of this game?

Roll	1, 2, or 3	$\frac{3}{6}=\frac{1}{2}$	+10
Roll	4 or 5	$\frac{2}{6}=\frac{1}{3}$	-13
Roll	6	$\frac{1}{6}$	-1

$(\frac{1}{2})(10) + (\frac{1}{3})(-13) + (\frac{1}{6})(-1)$

$= \frac{1}{2}$ point / game

Ex 4: In Monte Carlo, the game of roulette is played on a wheel with slots numbered 0, 1, 2, ..., 36. (37) The wheel spun and a ball dropped on the wheel is equally likely to end up in any one of the slots. To play the game, you bet \$1 on any number. If the ball stops in your slot, you win \$36 (the \$1 you bet plus \$35). Find the expected value of this game.

	Prob.	\$
win	$\frac{1}{37}$	36 − 1 = \$35
lose	$\frac{36}{37}$	-\$1

$(\frac{1}{37})(35) + (\frac{36}{37})(-1)$

$= -0.03$ ¢ / game

Lose 3¢ / game

Ex 5: A sweepstakes contest offers a first prize of one million dollars, a second prize of \$200,000, and a third prize of \$40,000. Suppose that three million people enter the contest and three names are selected randomly for the three prizes.

(a) What are the expected winnings of a person participating in this contest?

(b) Is it worth paying \$0.50 to enter this contest?

		\$
1st	$\frac{1}{3,000,000}$	+\$1,000,000
2nd	$\frac{1}{2,999,999}$	+\$200,000
3rd	$\frac{1}{2,999,998}$	+\$40,000

$$\left(\frac{1}{3,000,000}\right)(1,000,000) + \left(\frac{1}{2,999,999}\right)(200,00) + \left(\frac{1}{2,999,998}\right)(40,000)$$

$= 0.41$ ¢

No; expected value is less than entry

Ex 6: Real Life Ex: A life insurance policy for a 40-year old woman will pay \$10,000 if she dies within 1 year. The policy costs \$300. Statistics (namely, mortality tables) indicate that the relative frequency of a 40-year old woman dying within 1 year is 0.02. What is the expected profit of this policy to the woman?

not dying 0.98

dying	.02	10000 − 300 = \$9700
not dying	.98	− \$300

$(.02)(9700) + (.98)(-300)$

$= -\$100$

~~Ex 7: A game consists of drawing a card from a deck. You win \$13 if you draw an ace. What is a "fair price" to pay to play this game? ("Fair price" implies the price at which the player will break even, or in other words, the price at which expectation is zero).~~

Unit 7 Day 5 HW

1. A student plays the following game. He tossed three coins. If he gets exactly two heads he wins \$5. If he gets exactly one head he wins \$3. Otherwise, he loses \$2. On the average, how much should he win or lose per play of the game? (Use the word "win" or "lose" in your answer.

2. A detective figures that he has a $\frac{1}{9}$ chance of recovering some stolen property. He works on a contingency plan. He gets his money if he recovers the property but he does not get his money if he does not recover the property. The investigation costs will be \$9000. How large should his fee be so that, on average, his fee will be covered?

3. At Tucson Raceway Park, your horse, Stick-in-the-mud has a probably of $\frac{1}{20}$ of coming in first place, a probability of $\frac{1}{10}$ of coming in second, and a probability of $\frac{1}{4}$ of coming in third. First place wins \$4500, second place \$3500, and third place \$1500. It costs you \$1000 to enter the race. What is the expected value of the race to you? Is it worthwhile for you to enter the race? Explain.

4. A social club has a drawing every Friday night. The probability of winning the first prize of $100 is 0.002. The probability of winning the second prize of $80 is 0.01. How much should the club charge for tickets to enter the drawing so that the club breaks even?

5. You plan to invest in a certain project. There is a 35% chance that you will lose $30,000, a 40% chance that you will break even, and a 25% chance that you will make $55,000. What is the expected value in this problem, and what does it mean in terms of your investment?

6. A game consists of tossing a coin twice. A player who tosses two of the same face wins $1. How much should organizers charge to enter the game if they want to average $1.00 profit per person?

7. Consider a hat with pieces of paper inside. The papers are numbered as follows: 5 pieces with the number "1," 6 pieces with the number "7," and 9 pieces with the number "50." Find the expected value for drawing from this hat.

8. "Wheel of Fortune" just got a new wheel! On it there are 6 slots worth $200, 15 slots worth $400, 2 slots worth $600, 1 slot worth $1000, 6 slots with no money, and 1 slot with a car worth $20,000. What is the expected winnings on one turn(cash and prizes)?

9. In a game, you roll a die. If you get a 1 or a 5, you would win $5. If you roll a 4 you win $15 and if you roll a 2, 3, or 6 you lose $10. What is the expected value of one roll of the die?

10. A raffle is held by the MSUM student association to draw for a $1000 plasma television. Two thousand tickets are sold at $1.00 each. Find the expected value of one ticket.

11. A game consists of rolling a colored die with three red sides, two green sides, and one blue side. A roll of a red loses. A roll of green pays \$2.00. A roll of blue pays \$5.00. The charge to play the game is \$2.00. Would you play the game? Why or why not?

12. A company believes it has a 40% chance of being successful on bidding a contract that yields a profit of \$30,000. Assume it costs \$5,000 in consultant fees to prepare the bid. What is the expected gain or loss for the company if it decides to bid on the contract?

13. A department store wants to sell eight purses that cost the store \$40 each and 32 purses that cost the store \$10 each. If all purses are wrapped in forty identical boxes and if each customer picks a box randomly, find

(a) each customer's expected value if a customer pays \$15 for a box

(b) the department store's total expected profit (or loss) during this sale.

14. Assume that the odds against a certain horse winning a race are 5 to 2. If a better wins \$14 when the horse wins, how much should the person bet to make the game "fair"?