AFM **Name** Key

Unit 7 Day 7 Notes – Binomial Probability **Date** ____________

In a binomial experiment there are two mutually exclusive outcomes, often referred to as "success" and "failure".

- If the probability of success is ***p***, the probability of failure is ***1-p***.

Such an experiment whose outcome is random and can be either of two possibilities, "success" or "failure", is called a **Bernoulli trial**, after Swiss mathematician Jacob Bernoulli (1654 - 1705).

Examples of Bernoulli trials:

- flipping a coin -- heads is success, tails is failure
- rolling a die -- 3 is success, anything else is failure
- voting -- votes for candidate A is success, anything else is failure
- determining eye color -- green eyes is success, anything else is failure
- spraying crops -- the insects are killed is success, anything else is failure

When computing a **binomial probability**, it is necessary to calculate and multiply three separate factors:

1. the number of ways to select exactly r successes,
2. the probability of success (p) raised to the r power,
3. the probability of failure (q) raised to the (n - r) power.

The probability of an event, p, occurring exactly r times
$_nC_r \bullet p^r \bullet q^{n-r}$ n = number of trials r = number of specific events you wish to obtain p = probability that the event will occur q = probability that the event will **not** occur (q = 1 - p, the *complement* of the event)

Example:

1. A test consists of 10 multiple choice questions with five choices for each question. As an experiment, you GUESS on each and every answer without even reading the questions.
   What is the probability of getting exactly 6 questions correct on this test?

$n = 10$

$r = 6$

$p = 1/5$

$q = 4/5$

$$({}_{10}C_6)(1/5)^6 \cdot (4/5)^{10-6}$$

$$({}_{10}C_6)(1/5)^6 \cdot (4/5)^4 \approx .0055$$

$^{15}/_{50} = {}^{3}/_{10}$

2. At a certain intersection, the light for eastbound traffic is red for 15 seconds, yellow for 5 seconds, and green for 30 seconds. Find the probability that out of the next eight eastbound cars that arrive randomly at the light, exactly three will be stopped by a red light.

$({}_8C_3)(3/10)^3(7/10)^5 \approx .254$

3. When rolling a die 100 times, what is the probability of rolling a "4" exactly 25 times?

$({}_{100}C_{25})(1/6)^{25}(5/6)^{75} \approx .0098$

Note: When computing **"at least"** and **"at most"** probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
- all probabilities smaller than the given probability ("at most")

4. A bag contains 6 red Bingo chips, 4 blue Bingo chips, and 7 white Bingo chips.
What is the probability of drawing a red Bingo chip at least 3 out of 5 times? Round answer to the *nearest hundredth*.

To solve this problem, we need to find the probabilities that r could be 3 or 4 or 5, to satisfy the condition "at least".

It will be necessary to compute $\binom{n}{r} \cdot p^r \cdot (1-p)^{n-r}$
for $r = 3$, $r = 4$ and $r = 5$ and add these three probabilities for the final answer.
We need to compute:

For $r = 3$:	$({}_5C_3)(6/17)^3(11/17)^2 \approx .184$
For $r = 4$:	$({}_5C_4)(6/17)^4(11/17)^1 \approx .05$
For $r = 5$:	$({}_5C_5)(6/17)^5(11/17)^0 \approx .005$
Sum:	$.184 + .05 + .005 \approx .239$

Remember: "At most 2" is equivalent to "at least 5" when referring to 5 cases.

5. A family consists of 3 children. What is the probability that at most 2 of the children are boys?

Solution:

"At most" 2 boys implies that there could be 0, 1, or 2 boys. The probability of a boy child (or a girl child) is 1/2.

For $r = 0$:	$({}_3C_0)(1/2)^0(1/2)^3 = .125$
For $r = 1$:	$({}_3C_1)(1/2)^1(1/2)^2 = .375$
For $r = 2$:	$({}_3C_2)(1/2)^2(1/2)^1 = .375$
Sum:	$.125 + .375 + .375 = .875$

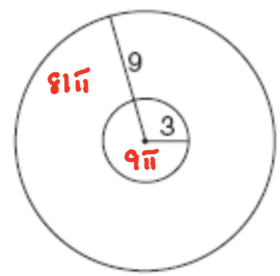
6. Team A and Team B are playing in a league. They will play each other five times. If the probability that team A wins a game is 1/3, what is the probability that team A will win at least three of the five games?

Solution:

"At least" 3 wins implies 3, 4, or 5 wins. $n=5 \quad p=1/3 \quad q=2/3$

For $r=3$:	$({}_5C_3)(1/3)^3(2/3)^2 \approx .164$
For $r=4$:	$({}_5C_4)(1/3)^4(2/3)^1 \approx .041$
For $r=5$:	$({}_5C_5)(1/3)^5(2/3)^0 \approx .004$
Sum:	$\approx .209$

7. As shown in the accompanying diagram, a circular target with a radius of 9 inches has a bull's-eye that has a radius of 3 inches. If five arrows randomly hit the target, what is the probability that at least four hit the bull's-eye? Express answer to the *nearest thousandth.*



Solution:

"At least" 4 hits implies 4 or 5 hits. The area of the bull's-eye is 9π *and the area of the entire target is* 81π. *The probability of hitting the desired bull's-eye is 1/9.*

not hitting bull's-eye $8/9$

For $r=4$:	$({}_5C_4)(1/9)^4(8/9)^1 \approx .00068$
For $r=5$:	$({}_5C_5)(1/9)^5(8/9)^0 \approx .000017$
Sum:	$\approx .000697$

Unit 7 Day 7 HW(1) #'s 9 + 10

Given the number of trials and the probability of success, determine the probability indicated: Set up problems 1-4 without the calculator.

1. $n = 12,\ p = 0.2,$ find P(2 successes)

2. $n = 15,\ p = 0.9,$ find P(11 successes)

3. $n = 7,\ p = \frac{1}{3},$ find P(4 successes)

4. $n = 15,\ p = 0.99,$ find P(1 failure)

5. $n = 6,\ p = 0.35,$ find P(at least 3 successes)

6. $n = 100,\ p = 0.01,$ find P(no more than 3 successes)

7. In a history class, Colin and Diana both write a multiple choice quiz. There are 10 questions. Each question has five possible answers. What is the probability that
   a) Colin will pass the test if he guesses an answer to each question.

   b) Diana will pass the test if she studies so that she has a 75% chance of answering each question correctly.

8. The manufacturing sector contributes 17% of Canada's gross domestic product. A customer orders 50 components from a factory that has a 99% quality production rate (99% of the products are defect-free). Find the probability that:

a) none of the components in the order are defective

b) there is at least one defective product in the order.

c) There are at least two defective products in the order.

9. A pair of dice is rolled 20 times. What is the probability that a sum of 5 is rolled

a) exactly 6 times

b) at least 4 times

c) at most 5 times

10. The probability the Tim will sink a foul shot is 70%. If Tim attempts 30 foul shots, what is the probability that

a) he sinks exactly 21 shots

b) he sinks at most 21 shots

c) he sinks between 18 and 20 shots, inclusive.

Unit 7 Day HW(2) – #'s 1, 2, + 3

1. Compute the probability of X successes, using the binomial formula.
   (a) $n = 5, X = 2, p = 0.025$
   (b) $n = 12, X = 6, p = 0.45$
   (c) $n = 6, X = 0, q = 0.35$
   (d) $n = 45, X = 10, p = 0.25$
   (e) $n = 22, X = 20, p = 0.68$

2. Compute the probability of X successes given $n = 12$ and $p = 0.45$ using the binomial formula.
   (a) $P(X = 6)$
   (b) $P(X \geq 9)$
   (c) $P(X<4)$
   (d) $P(4 <X<7)$
   (e) $P(5<X<7)$

3. A student randomly guesses at 10 multiple choice questions. Each question has four possible answers with only one being correct, and each is independent of every other question.
   (a) Find the probability that the student guesses EXACTLY 4 correct.
   (b) Find the probability of guessing less than 3 correctly.
   (c) Find the probability of guessing more than 8 correctly.
   (d) Find the probability of guessing between 4 and 6 inclusively.

4. In a Gallop Poll conducted January 30 – February 2, 2008, 43% of 18-29 year olds said that they were worried about retirement. Find the probability that out of 15 college students ages 18 – 19:

   (a) Exactly 1 worried about retirement.
   (b) Fewer than 5 worried about retirement.
   (c) At least 10 worried about retirement.
   (d) Between 8 and 10 inclusively are worried about retirement.

5. In a Gallop Poll, 35% of 30-49 year olds stated they believe in ghosts. Find the probability that out of 16 college students aged 30 – 49:

   (a) Exactly 5 said they believed in ghosts.
   (b) Exactly 5 said they do not believe in ghosts.
   (c) At least 4 believe in ghosts.
   (d) At least 4 do not believe in ghosts.