AFM
Unit 7 Test Review

Name Key
Date _____

Permutation: Order matters
nPr

Combination: Order doesn't matter
nCr

Distinguishable Permutations
$$\frac{(total\ items)!}{(repeat)!(repeat)!...}$$

Fundamental Counting Principle
*using all items, fill in blanks, multiply

Probability: Likelihood of an event occurring to total number of outcomes

Independent/Dependent (AND) vs. Mutually Inclusive/Exclusive (OR)

AND…MULTIPLY	OR…ADD
Independent One event does not affect the outcome of the second event Ex: Flipping a coin and rolling a die P(A) x P(B)	**Mutually Exclusive** The events cannot happen at the same time Ex: Being a boy vs being a girl P(A)+P(B)
Dependent One event affects the outcome of the second event Ex> picking a card and picking a second card without replacing the first card P(A) x P(B) (after A happens)	**Mutually Inclusive** The events can happen at the same time Ex: Being a boy and having blue eyes P(A)+ P(B) – P(A and B)

Binomial Probability
(exactly, at least, at most)

The probability of an event, p, occurring exactly r times
$${}_nC_r \cdot p^r \cdot q^{n-r}$$ n = number of trials r = number of specific events you wish to obtain p = probability that the event will occur q = probability that the event will **not** occur (q = 1 - p, the *complement* of the event)

Expected Value
Make a table like this…

Outcome	Probability	Value

E=(outcome)(probability)+(outcome)(probability)+…
*Don't forget the amount you pay to play is a negative *All probability adds up to 1

Geometric Probability
$$\boldsymbol{Probability = \frac{Area\ of\ Desired\ Region}{Total\ Area}}$$

Circle: $A = \pi r^2$

Square/Rectangle: $A = l \cdot w$

Triangle: $A = \frac{1}{2}bh$ or $A = \frac{1}{2}absinC$

Counting & Probability Review Problems

1. Three fair coins are tossed. What is the probability that there is only one head?

HHH THH THT
HHT TTH TTT
HTT HTH

$\frac{3}{8}$

2. A poker hand consisting of five cards is dealt from a deck of 52 cards. Find the probability of all five cards being spades.

$\frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2598960} = 4.952 \times 10^{-4}$ or 0.0004952

3. A jar contains 5 red, 4 green balls and 2 yellow balls. Find the probability of the given event.
   a. A green ball is drawn and replaced and then a yellow ball is drawn.

$\left(\frac{4}{11}\right)\cdot\left(\frac{2}{11}\right) = \frac{8}{121}$ or $.066$

   b. A red ball is drawn and then a white ball is drawn without replacing the firs ball.

$\left(\frac{5}{11}\right)\cdot\left(\frac{0}{10}\right) = 0$

   c. A red ball is drawn and then another red ball is drawn without replacing the first ball.

$\left(\frac{5}{11}\right)\cdot\left(\frac{4}{10}\right) = \frac{2}{11}$ or $.\overline{18}$

4. A researcher claims that she has taught a monkey to spell the word LEOPARD using six wooden letters D, A, R, L, O, P, and E. If the monkey has not actually learned anything and is merely arranging the blocks randomly, then
   a. What is the probability that he will spell the word correctly

$\frac{1}{7!} = \frac{1}{5040}$

   b. What is the probability that he will spell the word correctly three consecutive times?

$\left(\frac{1}{5040}\right)^3 = 7.811 \times 10^{-12}$

5. A card is drawn, a die is rolled and a coin is tossed. Find the probability of each outcome.
   a. The queen of hearts, a two and a tails,

$\left(\frac{1}{52}\right)\cdot\left(\frac{1}{6}\right)\cdot\left(\frac{1}{2}\right) = \frac{1}{624}$ or $.0016$

   b. A face card, a number more than three, and a heads.

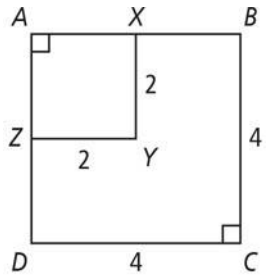
$\left(\frac{12}{52}\right)\cdot\left(\frac{3}{6}\right)\cdot\left(\frac{1}{2}\right) = \frac{3}{52}$ or $.0577$

6. ~~In our class, what is the probability of picking a student that is male or picking a student that has blonde hair?~~

7. You are taking a true/false quiz with 9 questions. What is the probability of getting all 9 questions correct?

$\frac{1}{2^9} = \frac{1}{512}$

8. Point P in square ABCD is chosen at random. Find the probability that point P is in squatter AXYZ.



Area smaller □ $= 2 \cdot 2 = 4$

Area larger □ $= 4 \cdot 4 = 16$

$\frac{4}{16} = \frac{1}{4}$ or .25

9. The probability of a student passing my class is 0.8. If 5 students are selected at random, what is the probability that at least 4 students will pass the class.

$({}_5C_4)(.8)^4(.2)^1 = .4096$

$({}_5C_5)(.8)^5(.2)^0 = .32768$

$.4096 + .32768 = .73728$

10. At the State Fair there is a booth where people can throw dimes onto a table that has dishes on it. Suppose that the chance that a dime lands in a dish is 0.3. Suppose you play the game 10 times.
    a. What is the probability that you will throw a dime in a dish at least 8 times?

$({}_{10}C_8)(.3)^8(.7)^2 = .00145$

$({}_{10}C_9)(.3)^9(.7)^1 = .000014$

$({}_{10}C_{10})(.3)^{10} = .0000059$

$.00145 + .000014 + .0000059 = .0028559$

    b. What is the probability that you will throw a dime in the dish exactly 7 times?

$({}_{10}C_7)(.3)^7(.7)^3 = .009$

11. You won a trip to Vegas and are feeling pretty lucky! You decide to play a game where you roll a die 8 times. What is the probability that
    a. You roll a number greater than 4 at most 2 times?

$({}_8C_2)(1/3)^2(2/3)^6 = .2731$

$({}_8C_1)(1/3)^1(2/3)^7 = .1561$

$({}_8C_0)(1/3)^0(2/3)^8 = .039$

$.2731 + .1561 + .039 = .4682$

    b. You roll an even number exactly 3 times?

$({}_8C_3)(.5)^3(.5)^5 = \frac{7}{32}$ or .21875

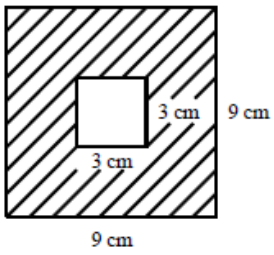
    c. You roll a 6 at least 7 times?

$({}_8C_7)(1/6)^7(5/6)^1 = .0024$

$({}_8C_8)(1/6)^8(5/6)^0 = .0001$

$.0024 + .0001 = .00241$

12. Find the probability of landing in the shaded region.

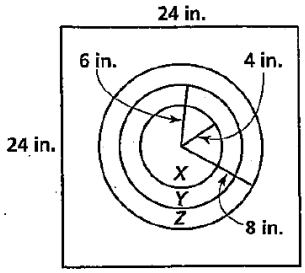


$\text{Area}_{\text{smaller square}} = 3 \cdot 3 = 9$

$\text{Area}_{\text{larger square}} = 9 \cdot 9 = 81$

$\frac{9}{81} = \frac{1}{9}$

13. Use the following dart board to answer the questions below:



$\text{Area}_{\text{square}} = 576 \text{ in}^2$

$\text{Area}_X \approx 50 \text{ in}^2$

$\text{Area}_Y \approx 113 \text{ in}^2$

$\text{Area}_Z \approx 201 \text{ in}^2$

a. If a dart hits the board, find the probability that it will land in region X

$\approx .087$

b. If a dart hits the board, find the probability that it will land in region Z

$\approx .349$

c. If a dart hits the board, find the probability that it will NOT hit any of the circles.

$\approx .368$

14. Three digit numbers are formed using the digits 2, 4, 5 and 8, with repetition of digits allowed. How many such numbers are there that are:

less than 800?

D_1	D_2	D_3
3	4	4

48 #'s

even?

D_1	D_2	D_3
4	4	2

32 #'s

15. A fancy restaurant offers 6 appetizers, 4 types of salads, 12 main courses and 3 desserts. In how many different ways can a customer order a meal from this restaurant?

864 meals

16. Mr. Davis has 24 people, 16 female and 8 male, trying out for the school play. He wants to choose a leading male, a leading female, a supporting male, supporting female and 6 extras - 2 males and 4 females. In how many ways can the cast be chosen?

$({}_8P_2) \cdot ({}_{16}P_2) \cdot ({}_6C_2) \cdot ({}_{14}C_4)$

$= 18345600$

17. Art, Becky, Carl, Denise, and Ed all want to go see Carrie Underwood in concert. However, they only have 3 tickets. How many ways can they choose the three who get to go to the concert?

$_5C_3 = 10$

18. A volleyball team has nine players. In how many ways can a starting line-up be chosen consisting of two forward players and three defense players?

$(_9P_2) \cdot (_7P_3) = 15120$

19. Part I of an exam has 5 multiple choice questions with 4 choices for each question. In how many different ways can this part of the exam be completed?

$4^5 = 1024$

20. Refer to question 6, assume you randomly guess the answers at the answers. What is the probability that you do not get all answers correct?

$\frac{1}{1024}$ ALL Correct $\qquad \frac{1023}{1024}$ None Correct

21. A child's game has a spinner which has spaces labeled 1 to 9 and all of the spaces are of equal size. What is the probability that the spinner stops on an odd number or a number greater than 6?

1 2 3 4 5 6 7 8 9

$\left(\frac{5}{9}\right) + \left(\frac{3}{9}\right) - \left(\frac{2}{9}\right) = \frac{6}{9} = \frac{2}{3}$ or $.\overline{6}$

22. A bowl contains 5 oranges and 4 tangerines. Noelle randomly selects one, puts it back, and then selects another. What is the probability that both selections are tangerines?

$\left(\frac{4}{9}\right) \cdot \left(\frac{4}{9}\right) = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$ or .198

23. A ball is randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. Find the probability that the ball is red or yellow.

$\left(\frac{5}{9}\right) + \left(\frac{1}{9}\right) = \frac{6}{9} = \frac{2}{3}$ or $.\overline{6}$

24. Two balls are randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. If the ball is not replaced after the first is selected, find the probability that both are red.

$\left(\frac{5}{9}\right) \cdot \left(\frac{4}{8}\right) = \frac{5}{18}$ or $.2\overline{7}$

25. You pay \$0.50 to draw one card from a deck of cards. If it is an ace, you win \$10; if it is a face card, you win \$1; otherwise, you lose. What is the expected value of this game? Is this game fair?

ace	1/13	\$9.50
face card	3/13	\$.50
other	9/13	-\$1

$(1/13)(9.50) + (3/13)(.50) + (9/13)(-1)$
$= .15¢$

26. A \$1 bet is made to draw three cards from a standard deck of 52 cards. If all three cards are face cards (12 face cards in a deck), then you win \$4.00. Find the expectation of this game and explain if you should play or not.

3 aces	.0099	\$3
other	.783	-\$1

$(.0099)(3) + (.783)(-1)$
$= -.75¢$ No

27. Suppose you surveyed the students in your class on their favorite juice flavors. Their choices were 6 apples, 10 orange, 1 grapefruit, and 3 mangos. Record the *experimental* probability for each flavor.

A: $4/20 = 3/10$
O: $10/20 = 1/2$
G: $1/20$
M: $3/20$